



**CHAPTER 9 EXERCISES**
 **Challenge**
 **Discussion**
**9.1 Majority Rule and Condorcet's Method**

- In a few sentences, explain why minority rule (the voting procedure for two alternatives that is described on page 287) satisfies conditions (1) and (2) on page 287, but not (3).
- In a few sentences, explain why imposed rule (the voting procedure for two alternatives that is described on page 287) satisfies conditions (1) and (3) on page 287, but not (2).
- In a few sentences, explain why a dictatorship (the voting procedure for two alternatives that is described on page 287) satisfies conditions (2) and (3) on page 287, but not (1).
- Find (or invent) a voting rule for two alternatives that satisfies
  - condition (1) on page 287, but neither (2) nor (3).
  - condition (2) on page 287, but neither (1) nor (3).
  - condition (3) on page 287, but neither (1) nor (2).
- In a sentence or two, explain why it's impossible, with an odd number of voters, to have two distinct candidates win the same election using Condorcet's method.
- Construct a real-world example (perhaps involving yourself and two friends) where the individual preference lists for three alternatives are as in the voting paradox of Condorcet.
- Condorcet's voting paradox shows that with three voters (or three equal-size groups of voters) and the three alternatives  $A$ ,  $B$ , and  $C$ , it is possible to have two-thirds prefer  $A$  to  $B$ , two-thirds prefer  $B$  to  $C$ , and two-thirds prefer  $C$  to  $A$ . Find four preference lists that show that with four voters and the four alternatives  $A$ ,  $B$ ,  $C$ , and  $D$ , it is possible to have three-fourths prefer  $A$  to  $B$ , three-fourths prefer  $B$  to  $C$ , three-fourths prefer  $C$  to  $D$ , and three-fourths prefer  $D$  to  $A$ .
- Generalize the result in Exercise 7 from four alternatives to  $n$  alternatives:  $A_1, \dots, A_n$ .

**9.2 Other Voting Systems for Three or More Candidates**

- Plurality voting is illustrated by the 1980 U.S. Senate race in New York among Alfonse D'Amato ( $D$ , a conservative), Elizabeth Holtzman ( $H$ , a liberal), and Jacob Javits ( $J$ , also a liberal). Reasonable estimates

(based largely on exit polls) suggest that voters ranked the candidates according to the following table:

	22%	23%	15%	29%	7%	4%
$D$	$D$	$D$	$H$	$H$	$J$	$J$
$H$	$J$	$J$	$D$	$J$	$H$	$D$
$J$	$H$	$H$	$J$	$D$	$D$	$H$

- Is there a Condorcet winner?
- Who won using plurality voting?

- (Everyone wins.) Consider the following set of preference lists:

Rank	Number of Voters (9)						
	3	1	1	1	1	1	1
First	$A$	$A$	$B$	$B$	$C$	$C$	$D$
Second	$D$	$B$	$C$	$C$	$B$	$D$	$C$
Third	$B$	$C$	$D$	$A$	$D$	$B$	$B$
Fourth	$C$	$D$	$A$	$D$	$A$	$A$	$A$

Note that the first list is held by three voters, not just one. Calculate the winner using

- plurality voting.
- the Borda count.
- the Hare system.
- sequential pairwise voting with the agenda  $A, B, C, D$ .

- Consider the following set of preference lists:

Rank	Number of Voters (7)				
	2	2	1	1	1
First	$C$	$D$	$C$	$B$	$A$
Second	$A$	$A$	$D$	$D$	$D$
Third	$B$	$C$	$A$	$A$	$B$
Fourth	$D$	$B$	$B$	$C$	$C$

Calculate the winner using

- plurality voting
- the Borda count.
- the Hare system.
- sequential pairwise voting with the agenda  $B, D, C, A$ .