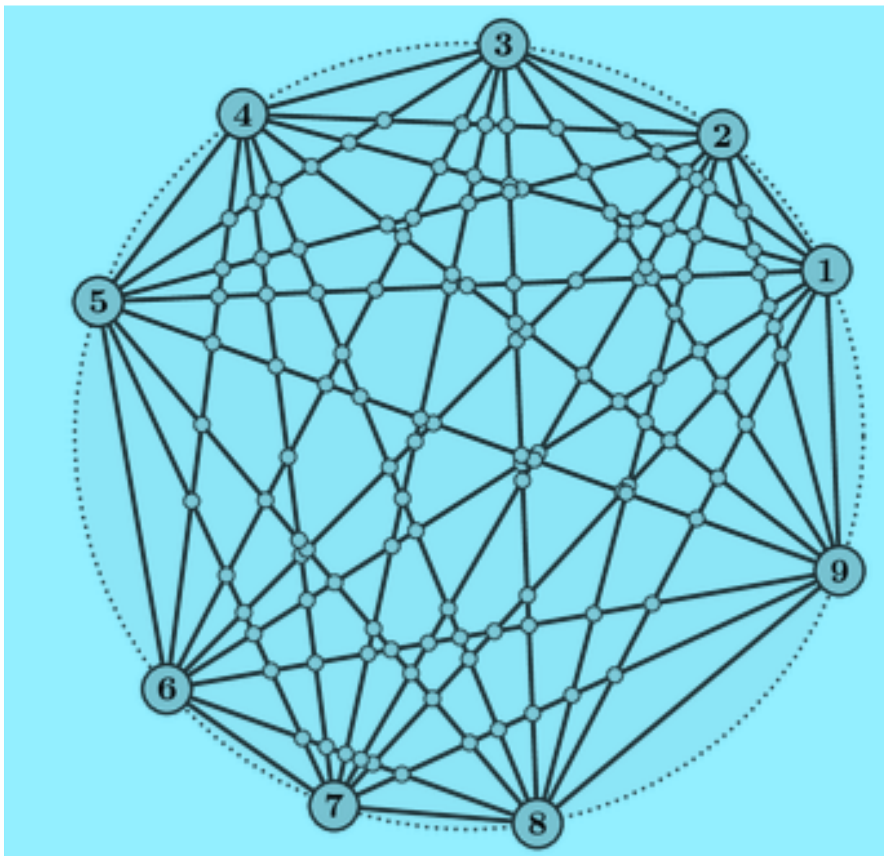
Discrete Mathematics Indicators



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PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education | Department of Public Instruction

Discrete Mathematics Indicators

Discrete Mathematics introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, and mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear and exponential functions to model and solve problems.
- Operate with matrices to model and solve problems.
- Define complex numbers and perform basic operations with them.

Strands: Number and Operations, Geometry & Measurement, Data Analysis and Probability, Algebra

COMPETENCY GOAL 1: The learner will use matrices and graphs to model relationships and solve problems.

Objectives

1.01 Use matrices to model and solve problems.

a) Display and interpret data.
b) Write and evaluate matrix expressions to solve problems.

1.02 Use graph theory to model relationships and solve problems.

COMPETENCY GOAL 2: The learner will analyze data and apply probability concepts to solve problems.

Objectives

2.01 Describe data to solve problems.

a) Apply and compare methods of data collection.
b) Apply statistical principles and methods in sample surveys.
c) Determine measures of central tendency and spread.
d) Recognize, define, and use the normal distribution curve.
e) Interpret graphical displays of data.
f) Compare distributions of data.

2.02 Use theoretical and experimental probability to model and solve problems.

a) Use addition and multiplication principles.
b) Calculate and apply permutations and combinations.
c) Create and use simulations for probability models.
d) Find expected values and determine fairness.
e) Identify and use discrete random variables to solve problems.
f) Apply the Binomial Theorem.

2.03 Model and solve problems involving fair outcomes:

a) Apportionment.
b) Election Theory.
c) Voting Power.
d) Fair Division.

COMPETENCY GOAL 3: The learner will describe and use recursively-defined relationships to solve problems.

Objective

3.01 Use recursion to model and solve problems.

a. Find the sum of a finite sequence.
b. Find the sum of an infinite sequence.
c. Determine whether a given series converges or diverges.
d. Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas.
e. Verify an explicit definition with inductive proof.

Discrete Mathematics Objective 1.01
Matrices

Vocabulary/Concepts/Skills:

- Row
- Column
- Dimensions
- Inverse
- Identity
- Scalar
- Transpose
- Vectors
- Matrix Operations
- Leslie Model
- Markov Chain
- Cryptography
- Cipher
- Code
- Game Theory
- Payoff matrix
- Saddle Point

Example 1: A breed of lizard has a life span of at most three years. A female lizard will usually produce her first offspring at six months of age and continue producing offspring until she dies. Only 20% of newborn lizards reach the age of six months. Of those who do, 70% reach the age of one year. Of the ones who reach one year, 90% reach the age of 18 months. Of those, 80% reach the age of two years and 50% of those reach two and one-half years. None of the lizards live past the age of three years.

The birth rates by ages in months are as follows:

Age	Birth Rate
0 - 6 months	0
6 - 12 months	1.5
12 - 18 months	1.9
18-24 months	2.6
24 -30 months	2.1
30 - 36 months	1.4

a. Find the population distribution after four years have passed (this is P8) and after seven years have passed.
b. Find the total population at each of these times.

Example 2: A video storeowner has found that the probability that a customer who rented a movie today will also rent a movie tomorrow is 35% while the probability that a customer who did not rent today, will rent tomorrow is 10%.

a. Write the transition matrix that represents this information.
b. If 853 out of his 8745 customers rented a movie on Monday night (7892 of his customers didn't rent one on Monday) about how many customers can he expect to rent a movie on Tuesday? About how many of his customers can he expect to rent a movie three weeks from Monday?

Example 3: Suppose you and your friend are playing a game in which you each hold up an even or odd number of fingers on the count of three. If you are both holding up an even number, you get three pennies. If you are both holding up an odd number, you have to pay your friend two pennies. If one of you is holding up an even and the other an odd, you have to pay your friend one penny. Construct a payoff matrix to represent this game. Determine if there is a saddle point and your best strategy for the game.

Example 4: Decode the message -40, 109, -66, 173, -55, 145, -38, 95 if the original coding matrix was $\begin{bmatrix} -3 & 5 \\ -1 & 2 \end{bmatrix}$. Your message will give you the name of a famous piece of art.

Example 5: A population of laboratory animals models the following birth and survival rates.

Age	Birthrate	Survival Rate
0-3	0.3	0.9
3-6	0.9	0.7
6-9	0.9	0.8
9-12	0.6	0

The following table displays the initial population:

Months	0-3	3-6	6-9	9-12
Number	10	11	8	4

a. How many newborn animals were there after four cycles?
b. What is the total population after two cycles?
c. When will the population reach 800?
d. What is the long-term growth rate?

Example 6: During a soccer season, referees are paid different rates for the different type of games. There are three types of games in a typical season: non-conference, conference, and playoff games. There are two referees for each game and schools only have to pay for home games. The information below is from three high schools with the same pay scale for referees.

High School	Home Games Nonconference	Home Games Conference	Playoff Games	Total Pay for Soccer Referees
Green River	5	7	2	$1806
Blue Creek	3	8	1	$1570
Black Lake	6	6	0	$1476

a. Write the above information into a matrix equation such that $Matrix\ A \cdot \begin{bmatrix} n \\ c \\ p \end{bmatrix} = Matrix\ B$
b. Find the $|A|$ and A^{-1}.
c. When seeking to find how much a referee is paid for the different game types, Maria tells the class that they needed to complete $B \cdot A^{-1}$. Julia disagrees and says it should be $A^{-1} \cdot B$. Mike says that since multiplication is commutative, they both are correct and will get the same answer. Who is correct and how do you know?
d. Find how much each referee is paid for each type of game.

Example 7: Without the aid of technology, what is element e_{32}, of the product of $G \cdot H$?

$$\boldsymbol{G} = \begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{5} & \mathbf{0.5} \end{bmatrix} \qquad \boldsymbol{H} = \begin{bmatrix} \mathbf{7} & \mathbf{-1} & \mathbf{0.5} \\ \mathbf{-5} & \mathbf{-3.5} & \mathbf{4} \end{bmatrix}$$

Discrete Mathematics Objective 1.02
Graph Theory

Vocabulary/Concepts/Skills:

- Conflict map
- Planar
- Edges
- Degree
- Paths
- Circuits
- Cycle
- Connected/Complete
- Trees
- Digraphs
- Adjacent
- Loops
- Minimum Spanning Tree
- Euler Circuits and Paths
- Hamiltonian Circuits and Paths
- Bipartite Graphs
- Chromatic Number of a Graph
- Kruskal's Algorithm
- Prim's Algorithm Optimization
- Routing Problems
- Traveling Salesman Problems
- Critical Paths
- Earliest/Latest
- Minimization
- Nearest Neighbor Method

Example 1: In scheduling committee meetings for school improvement, six different meetings need to be scheduled.

Committee	Adam	Betty	Carl	Don	Eddy	Frank	Gus	Hank	Inez
Calendar	X			X				X	X
Academics	X					X	X	X	
Sports		X				X			X
Music/Art		X	X						X
Neighbors		X		X	X		X	X	
Building			X		X	X	X		

a. Based upon the information shown about which faculty members need to attend which meetings, what is the fewest number of time slots which could be used to schedule these six meetings?
b. Which meeting could be scheduled with the Music/Art meeting?

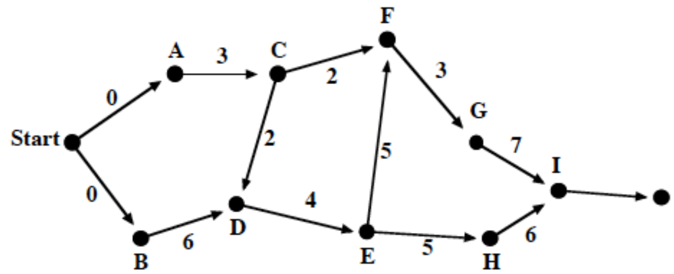
Example 2: Use the task list at right.
a. Find the earliest start time for each task.
b. Determine the minimum project time.
c. List the critical path.
d. Find the latest start time for task C.

Task	Time	Prerequisites
A	4	None
B	6	None
C	9	A
D	7	A, B
E	5	B
F	6	D
G	2	E, H
H	3	C, F
I	7	F
Finish		G, I

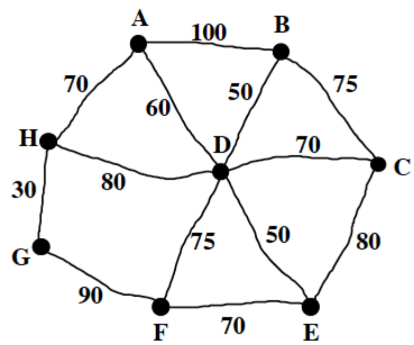
Example 3: Draw a tree with four vertices in which two vertices have a degree of three, one has a degree of two, and one has a degree of four.

Example 4: According to the graph shown

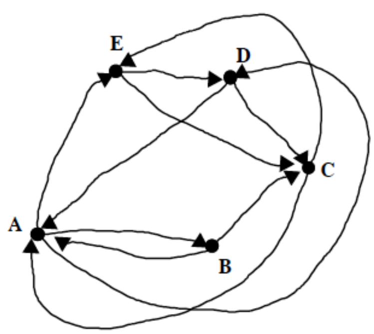
a. How many prerequisites does F have?
b. What is the earliest start time for E?
c. What is the minimum project time?
d. What is the critical path?



Example 5: Use Kruskal's algorithm to find the minimum spanning tree.



Example 6: Find an Euler circuit for the directed graph.

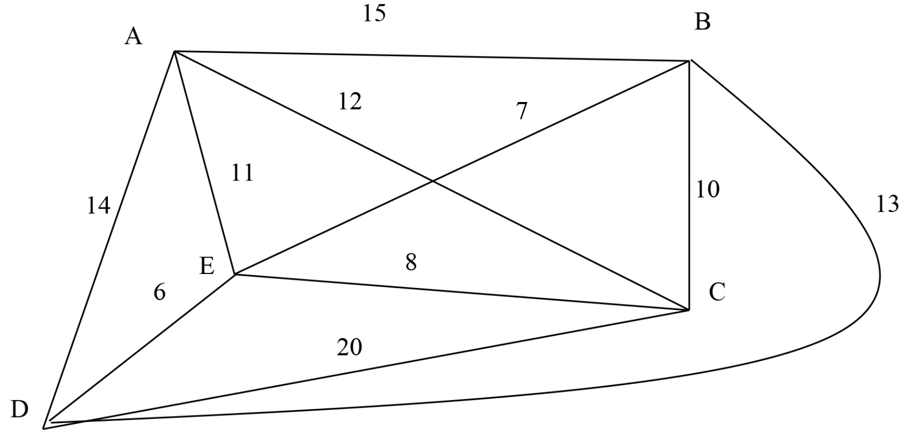


Example 7: A school district has five high schools, and they each play each other in one game of football. The county needs to rank their teams.
Based on the following information about the outcomes of the games, construct a digraph and then find the ranking of the teams in the tournament.

<u>**Outcomes:**</u>

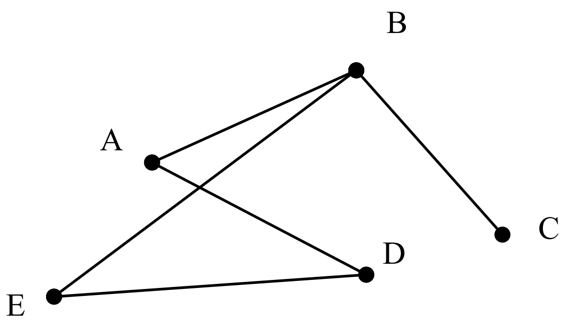
- The Bears beat the Wildcats.
- The Wildcats beat the Tigers.
- The Bulldogs beat the Bears, the Wildcats, and the Tigers.
- The Rams beat the Bears, the Wildcats, the Bulldogs, and the Tigers.

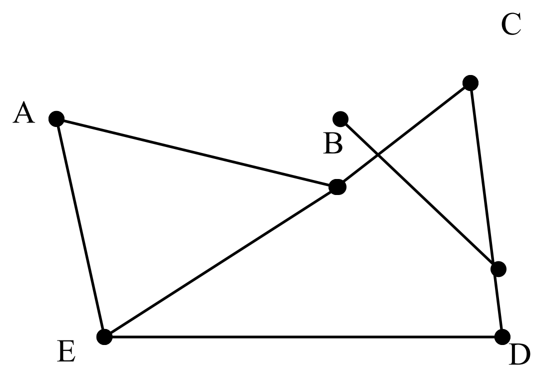
Example 8: Mr. Deal sells horse feed and must travel to four farms today and then return home. Since he pays for his own gas, he wants to insure that the distance he travels in total will be the least possible distance.



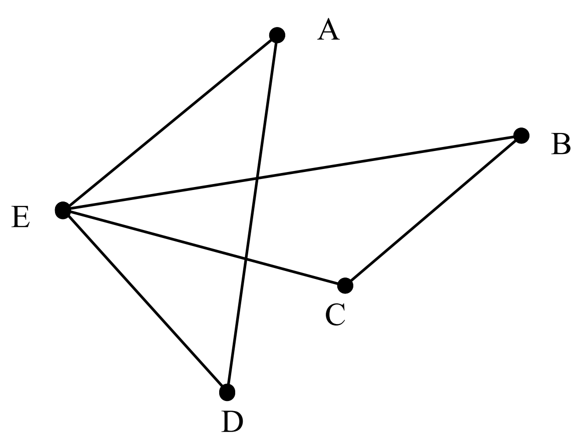
a. If his home is at **A**, find the best way for him to travel using the Brute Force Method and then the Nearest Neighbor Method.
b. Which method is always guaranteed to give you the shortest route?

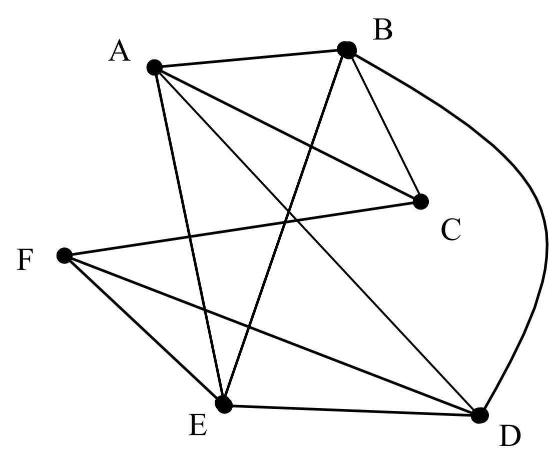
Example 9: Determine if the following graphs are bipartite. If so, list the two distinct sets of vertices.





Example 10: Determine if the following graphs have an Euler circuit, Euler path, both, or neither. State how you know this. Then determine if the graphs are planar. Finally, determine if the graphs have a Hamiltonian circuit.





Discrete Mathematics Objective 2.01
Describing Data

Vocabulary/Concepts/Skills:

- Mean
- Median
- Variance
- Z-Score
- Standard Deviation
- Normal Distribution
- Random Sampling
- Census
- Survey
- Bias
- Population
- Venn Diagrams
- Empirical Rule

Example 1: The averages of 28 students in a Geometry class and 25 students in a Discrete Math class are given below:

Geometry:
82, 100, 94, 68, 34, 72, 70, 96, 99, 92,
90, 85, 70, 46, 71, 71, 77, 78, 95, 82,
80, 100, 99, 72, 69, 74, 84, 87

Discrete:
100, 95, 72, 80, 97, 78, 89, 100, 93, 95, 66,
87, 85, 98, 89, 86, 80, 79, 94, 90, 92, 87, 88
81, 82

a. Calculate the mean, median, standard deviation, and interquartile range for each class.
b. Construct an appropriate graph to compare the two classes and then write several sentences to compare the class grades in context.

Example 2: 125 people have decided to take a trip. 46 people said they would be happy either going to New York City or Nashville. 38 people would be happy with Nashville or Orlando and 27 people would be happy with New York City or Orlando. 14 people only want to go to New York City, 8 people only to Nashville, and 12 people only to Orlando.

a. How many said they would be happy with any of the three places?
b. How many would be happy if Orlando were chosen?

Example 3: According to the CDC, girls aged two years have a mean height of 84.98 cm with a standard deviation of 1.73 cm.

a. What percent of two-year-old girls are taller than 86.71 cm?
b. What percent of two-year-old girls have heights between 81.52 cm and one standard deviation above the mean?
c. Between what two heights are the middle 99.7% of two-year-old girls?

Example 4: Human body temperatures are approximately normally distributed with a mean of 98.6 and standard deviation of .72.

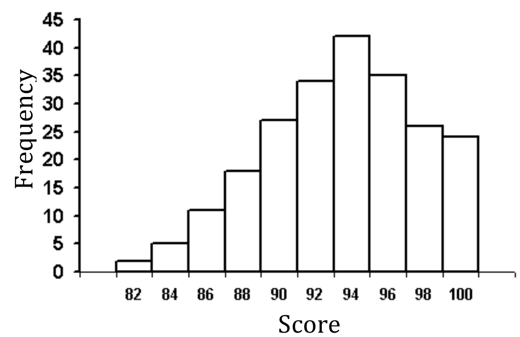
a. What percent of people have a body temperature between 97.88 and 99.32?
b. What percent of people have body temperatures below 97.88?
c. If you have a body temperature in the 84^{th} percentile, what is your body temperature?

Example 5: You are interested if students learn best in a math classroom with direct instruction and practice or in a blended learning situation in which students learn online and then have some teacher assistance when needed. To determine this, you come up with three different data collection methods you could try. State which would be an observational study, a survey, and an experiment.

a. You take a simple random sample of students in North Carolina and ask them questions about how their math instruction in provided and their scores on end of course exams, SAT, ACT, and classroom grades.
b. You randomly assign students to one teacher's 1st period and 2nd period Math III classes. The teacher must use direct instruction and practice in 1st period and blended learning in 2nd period. At the end of the semester you compare the class grades and end of course grades of the students in both classes.
c. You sit in various classrooms throughout the state, some with direct instruction and others with blended learning. You watch the classes and make notes as to how well the students seem to understand the material.

Example 6: The graph below shows the quiz scores on a Discrete Math quiz across an entire school.

a. Describe the data of this graph.
b. What is the relationship between the mean and the median for this data set?
c. Create a graphic that shows data that has the same mean and median.



Discrete Mathematics Objective 2.02
Probability

Vocabulary/Concepts/Skills:

- Counting
- Combination
- Permutation
- Factorial
- Event
- Success
- Trial
- Sample Space
- Mutually Exclusive
- Disjoint
- Dependent/Independent
- Binomial Probability
- Expected Value
- Random Variable
- Simulation
- Monte Carlo
- Empirical Probability
- Theoretical Probability

Example 1: You were invited to attend the Kentucky Derby next year. There will be 20 horses racing, but you don't know anything about any of the horses. You want to try to choose the top 4 horses in the correct order.

a. If every possibility of the top four horses is the same, what is the probability that you are correct?

Example 2: A survey was conducted of 1000 high school students regarding their gender and favorite subject in school. The results are as listed below:

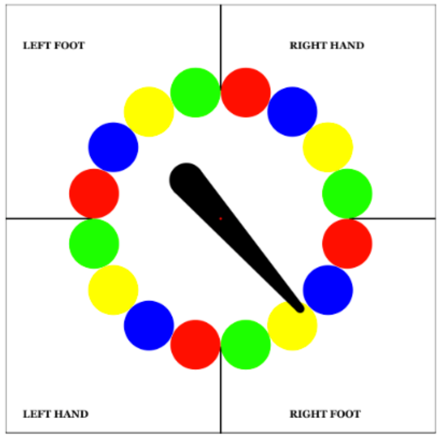
	English	Math	Science	Social Studies	Total
Male	30	78	159	271	538
Female	192	108	131	31	462
Total	222	186	290	302	1000

a. Find the probability of randomly choosing a student whose favorite subject is Science.
b. Find the probability of randomly choosing a student who is a female and whose favorite subject is English.
c. Find the probability of randomly choosing a male student whose favorite subject is Social Studies.
d. Determine if the events Female and Math are independent. Show that this is true.
e. Give an example of two events in the table that are mutually exclusive (disjoint) and two events that are not mutually exclusive.

Example 3: You draw three cards from a standard deck of cards without replacement.

a. Determine the probability of drawing two cards of one color and one card of another color.

Example 4: The game Hands & Feet uses a four-section spinner similar to the image below. There are four different colors that can be spun for each hand and foot.



a. What is the probability that you spin right hand exactly three out of ten times?
b. You want to make a new "Super" Hands & Feet game including two more colors and using HEAD as another body part. How many more spinning combinations are there now than in the original game? Note: An example of a spinning combination is left hand on red.

Example 5: A game of chance at the State fair lets a person choose a number between one and six and then roll two dice. They must pay one dollar to play. They win one dollar if their number appears on one die and two dollars if their number appears on two dice. They lose one dollar if their number doesn't appear at all.

a. If you play the game 100 times, how much money should you expect to win or loss?

Example 6: A football player is excited to know if he will start at the next game. The coach needs to selection one quarterback out of the three available, three receivers from the ten available, one running back from the four available, and six linemen from the remaining 33 players for a total of 11 players on offense.

a. Determine how many ways a coach can choose the starting offensive players on his football team.

Example 7: Each day, two out of three teams are randomly selected to participate in a game. What is the probability that team A is selected on at least two of the next three days?

Example 8: Mrs. Ormond notices that the students' test performance is affected by how well or how poorly they did on the last test. Specifically, 75% who did well on the last test do well on the next test, 20% do average, and 5% do poorly. Of the average scorers on the last test, 70% score average on the next test, 10% do well, and 20% do poorly. For those who did poorly on the last test, 86% do poorly on the last test, 10% do average, and 4% do well.
If Billy did average on the first test, how likely is he to do well on the fourth test? (That's three after the first one.)

Example 9: In a literature class, the students are going to be randomly assigned novel to read and write a report. The randomly assigned novel comes out of the teacher's library of 42 novels.

a. In a class of 27 students, how many combinations of novels and students are possible?
b. Before the books are assigned, the teacher tells the students that they will all get an A if anyone in the class can predict correctly which students are assigned which novel with the limit that each student only gets one guess. What is the probability of this occurring?
c. How could this problem have been rewritten so that order does not matter?

Example 10: A teacher is giving a 7 question true-false quiz. Some of the students were not prepared for the quiz and wanted to know what the probability was for a student to randomly guess at least 5 of the questions correctly to get a passing grade.

a. Design a simulation using appropriate technology and complete the chart below. Complete 30 trials.

Use http://studenthelp.cpm.org/m/TI-84/l/95350-ti-84-generating-random-numbers to assist with running a simulation using a random number generator on a TI.

Number of correct answers	Tally Marks	Total
0		
1		
2		
3		
4		
5		
6		
7		

b. Based on your simulation, what is the probability for a student to randomly guess at least 5 of the 7 questions correctly?
c. Now compare your results with other students. How can you improve upon the results of the simulation?

Example 11: For the following situations determine if empirical, theoretical, or both probabilities could be used and explain your choice.

a. Probability of a basketball player making a free throw
b. Probability of rolling a sum of 5 with two number cubes
c. Probability of winning the lottery
d. Probability of guessing the genre of the next song on your playlist

Discrete Mathematics Objective 2.03
Apportionment

Vocabulary/Concepts/Skills:

- Ideal Ratio
- Hamilton Method
- Jefferson Method
- Webster Method
- Hill Method
- Quota
- Violation of Quota (Quota Rule)
- Fair Apportionment Method

State	Population
Aram	5576
Bethel	1387
Capernaum	3334
Damascus	7513
Elam	311
Total	18121

Example 1: A very small country has five states and needs to apportion their legislature of 93 members according to their respective populations.

a. Apportion the 93 representatives according to the methods of Hamilton, Jefferson, Webster and Hill.

Example 2: Suppose there is a state with six counties (A, B, C, D, E, and F). There are 2,500,000 people in the state, and the state will have 50 elected representatives in their legislative branch of government. The representatives must be split up fairly among the counties, based on population.

a. Given the following populations, apportion the representatives using Hamilton's Method, Jefferson's Method, Webster's Method, and Hill's Method:

A: 329,200
B: 1,387,200
C: 30,800
D: 418,200
E: 137,000
F: 197,600

Example 3: Your high school must decide how many teachers to assign to each department. If your school has been allotted 40 teachers, demonstrate how to fairly distribute the teachers to each department based on the number of students who signed up for classes within that department.

a. Use Hamilton's Method, Jefferson's Method, Webster's Method, and Hill's Method to apportion the teachers.

English: 1959
History: 1525
Science: 1648
Math: 2068

Discrete Mathematics Objective 2.03
Election Theory

Vocabulary/Concepts/Skills:

- Plurality
- Majority
- Borda Count
- Runoff (plurality with runoff)
- Sequential Runoff (plurality with elimination)
- Condorcet
- Approval Voting
- Arrow's Condition
- Pairwise Comparison Method

Example 1: Use the preference schedules above to find the Borda count winner.

a. If B decides to withdraw, who will be the plurality winner?

A	B	C	A	D
B	C	D	C	A
C	A	B	B	C
D	D	A	D	B
16	**12**	**13**	**4**	**8**

Example 2: The prom committee at your school is trying to decide on a location to hold prom: Kingtime Suites, Horton Hotels, Sleepy Times, the Country Club, or your school. Students were asked to rank their preferences of locations from first to fifth, and 550 students voted. Their preference schedules are listed below:

Number of Students	**180**	**120**	**100**	**90**	**40**	**20**
1st Choice	Kingtime Suites	School	Country Club	Horton Hotels	Sleepy Times	Sleepy Times
2nd Choice	Horton Hotels	Sleepy Times	School	Country Club	School	Country Club
3rd Choice	Sleepy Times	Horton Hotels	Sleepy Times	Sleepy Times	Horton Hotels	Horton Hotels
4th Choice	Country Club	Country Club	Horton Hotels	School	Country Club	School
5th Choice	School	Kingtime Suites	Kingtime Suites	Kingtime Suites	Kingtime Suites	Kingtime Suites

a. Which location wins if the plurality method is used? Is this location also a majority winner?
b. Find which location would be the winner if the prom committee uses a 5, 4, 3, 2, 1 Borda Count. Which location wins if they use a 10, 8, 5, 2, 1 Borda Count?
c. Which location wins if a runoff election is held between the top two locations? Which one wins if the prom committee uses sequential runoff?
d. Which location would win if the prom committee uses the Condorcet method?
e. Which location wins if approval voting is used and every like their top three choices?

Discrete Mathematics Objective 2.03
Voting Power

<u>Vocabulary/Concepts/Skills:</u>

- Dictator
- Dummy
- Weighted Voting
- Shapley-Shubik Power Index
- Banzhaf Power Index

Example 1: Find the Shapley-Shubik Power Index for each voter.

$$[q: A, B, C] = [10: 3, 7, 9]$$

Example 2: Suppose the sophomore, junior, and senior class officers are voting on an issue. April, the senior class, officer has 9 votes; Ben, the junior class officer, has 6 votes; and Candace, the senior class officer, has 2 votes.

a. Find the Banzhaf power index for each individual.
b. Determine if there are any dummies and if there is a dictator.

Example 3: In the following voting situation, determine the Banzhaf power index. Will everyone feel as if they have equal power, or will one person feel as if he/she has more power? Why?

[6: 5, 3, 3]

Discrete Mathematics Objective 2.03
Fair Division

<u>Vocabulary/Concepts/Skills:</u>

- Desired Outcome
- Perceived Value (Fair Share)
- Divider/Chooser
- Marker Method
- Estate Division (Sealed Bids)

Example 1: Ann, Bob, Carol, and Don are the four heirs of an estate that includes a small house, a cabin in the mountains, a 1965 mustang and $95,000 in cash. They decide to divide the estate using sealed bids. Their bids on the three main items are listed in the following table. Each one is entitled to one-fourth of the estate.

Bids on:	**Ann**	**Bob**	**Carol**	**Don**
House	66,000	70,000	72,000	68,000
Cabin	42,000	37,000	36,000	40,000
Mustang	6,000	6,500	4,000	5,200

a. What does Ann receive as her share of the estate?
b. What does Bob receive as his share of the estate?
c. What does Carol receive as her share of the estate?
d. What does Don receive as his share of the estate?
e. Observe the situation from Bob's perspective. What is the dollar value of the share of each heir from Bob's view?

Example 2: Aunt Marge died recently and left a 1965 Porche, a diamond necklace, a condominium, and $10,500 in cash to her three nieces, Donna, Toni, and Linda. They decided to bid on each item.

	Donna	**Linda**	**Toni**
Porsche	3,000	2,800	5,000
Necklace	800	3,000	2,100
Condo	80,000	83,000	79,000

a. Find the final settlement for each person.

Example 3: Suppose you ordered a cookie cake that is half peanut butter cookie and half chocolate chip cookie. You are sharing with your two co-workers, Ann and Michael. You are going to use the Divider Chooser method, and you cut first so that one half of the cookie cake is 1/3 peanut butter and 2/3 chocolate chip. Before you cut the cookie, you did not know that Ann is allergic to peanuts and that Michael is allergic to chocolate. You just love all kinds of cookie cake.

a. If Ann is the first chooser, which half will she choose?
b. Describe how Ann will cut her piece into three pieces she feels are equal to her. Describe how you will cut your piece into three pieces that you feel are equal to you.
c. Which piece of Ann's will Michael take? Which piece of yours will Michael take?
d. Draw the cookie and how it has been cut. Then label each piece with the name of the person who will take it.
e. If you paid $9 for the cake, how much value did you put on the original half you cut? How much value did Ann put on the first "half" that she chose? How much value did Michael put on the first "half" that Michael chose?

Discrete Mathematics Objective 3.01
Recursion

Vocabulary/Concepts/Skills:

- Arithmetic Sequence
- Geometric Sequence
- Close Form
- Growth and Decay
- Sequences
- Series
- Inductive Proof

Example 1: For the following sequence: 5.3, 9, 12.7, 16.4, 20.1, 23.8, ...

a. Determine whether the sequence is arithmetic, geometric, mixed or something else.
b. Determine a recursive definition that will generate the sequence.
c. Determine a closed form function that will generate the sequence.
d. Find the value of the tenth term in the sequence.

Example 2: A recurrence relation is described by the equation $t_n = 3t_{n-1} - 7$.

a. What is the "fixed point" for this relation?

Example 3: Find the sum of the infinite geometric series shown: $54 + 18 + 6 + 2 + \cdots$

Example 4: Sarah puts $300 in a savings account that earns 5.25% compounded annually.

a. Write a recursive relation for the situation.
b. When will Sarah double her money?

Example 5: Find the 20th term of the sequence $10, 15, 22.5, 33.75, 50.625, \ldots$

Example 6: An auditorium has 18 seats in the first row. Each successive row has two additional seats. The last row has 84 seats.

a. Write a recursive relation for the number of seats in the nth row.
b. Write a closed form solution for the number of seats in the nth row.
c. How many rows are there?

Example 7: Write a recursive relation for the number of edges (E_n) in a complete graph with *n* vertices.