Euler Circuit Activities

Activities # 1, 2 & 3

Goal: To discover the relationship between a graph's valence and connectedness and how these factors impact whether it has an Euler circuit.

Key Words: Graph, vertex, edge, path, circuit, valence, Euler circuit, connected

Activity # 4

Goal: To learn the method of Eulerizing a circuit.

Key Words: Eulerizing a circuit

Euler Circuits

Name: ______________________________

Activity #1 – *Basic Definitions.*

Airline Example:

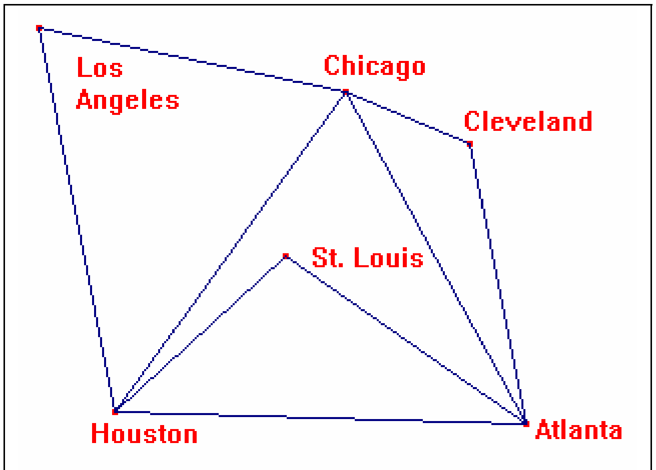

Figure 1

Graph – A finite set of dots and connecting links; suppose figure 1 is a graph representing airline flights available throughout the United States

Vertex – A dot on a graph; Cleveland is a vertex in figure 1

1. List the other vertices of figure 1.

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Edge – An edge is a link between two vertices; there is an edge connecting Chicago and Atlanta in figure 1

2. List the other edges in figure 1.

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Path – a connected sequence of edges showing a route on a graph that starts at a vertex and ends at a vertex; there is a path from Los Angeles to Cleveland through Chicago

3. How many paths can you find from Los Angeles to Cleveland? List them below.

Circuit – a path that starts and ends at the same vertex; there is a circuit from Cleveland to Atlanta to Chicago and ending in Cleveland

4. How many other circuits can you find beginning and ending in Cleveland with at most 4 stops? List them below.

Information Flow Example:

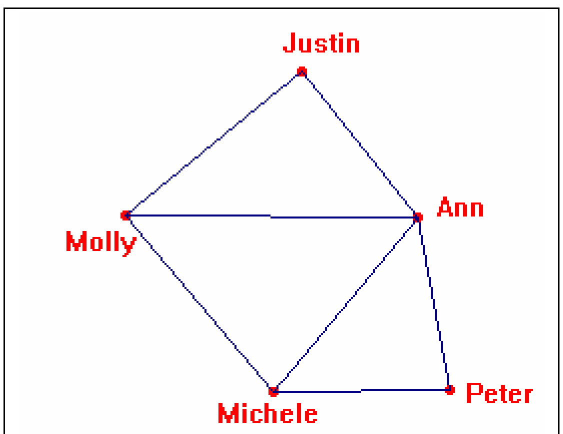

Figure 2

Suppose Figure 2 represents the flow of information within a company. Use this graph to answer the following questions.

1. List the vertices of figure 2.

2. List the edges of figure 2.

3. How many different paths can you find from Justin to Michele? List them below.

4. How many ways can information pass from Molly to Ann with at most three people between them? List them below.

Activity #2 – Euler Circuits and Valence:

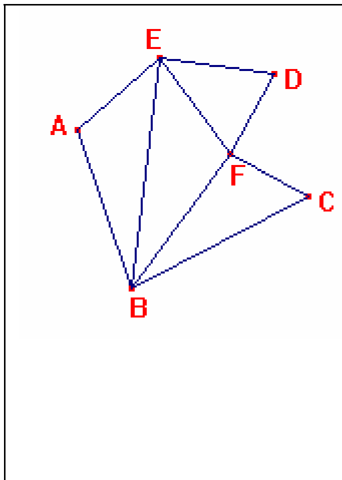

Figure 2

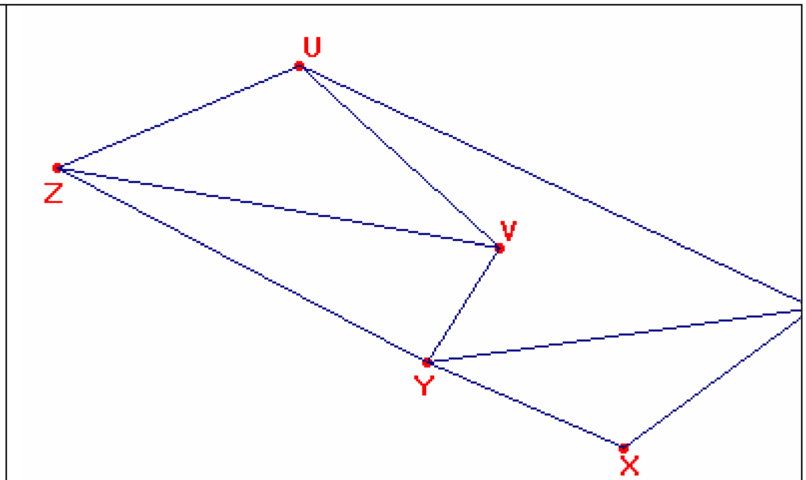

Figure 3

1. The ***valence*** of a vertex in a graph is the number of edges meeting at that vertex. Label the valences of each vertex in figures 2 and 3.

2. An ***Euler circuit*** is a path that begins and ends at the same vertex and covers every edge only once passing through every vertex. Refer back to the airline example. Can you trace an Euler circuit in figure 1 beginning and ending in Los Angeles? If so, describe your path below.

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3. Does figure 2 have an Euler circuit beginning and ending at D? Do the vertices in figure 2 have even or odd valences?

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4. Does figure 3 have an Euler circuit beginning and ending at V? Do the vertices in figure 3 have even or odd valences?

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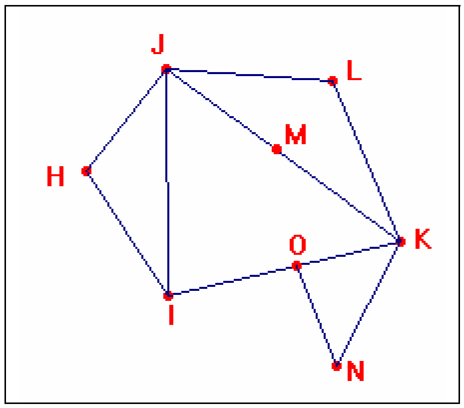
5. Based on your answers in questions 3 and 4, what property can you conjecture about a graph that has an Euler circuit?

6. Test your conjecture in the following graph by first finding the valence of each vertex, and then based on this guessing whether the graph has an Euler circuit or not. Write your reasoning and conclusion in the space provided.



7. In your own words, what is one condition that causes a graph to lack an Euler circuit?

Extension:
Based on the knowledge learned above, can you think of any practical uses of Euler circuits?

Activity #3 – Euler Circuits and Connectedness:

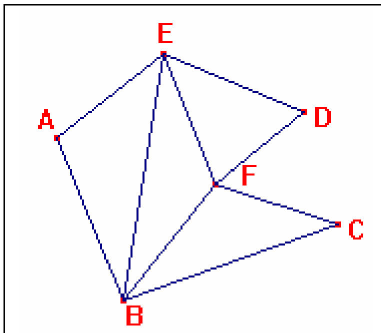


Figure 4

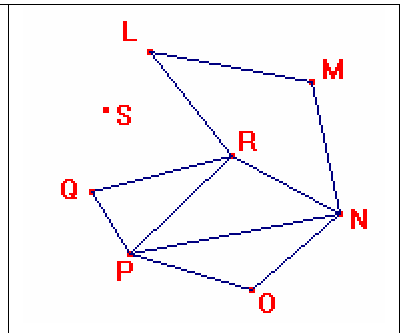


Figure 5

1. A graph is ***connected*** if you can always find a path between any pair of vertices. Refer to figure 4. Is the graph connected? Explain.

2. Can you trace an Euler circuit in figure 4? Remember that an Euler circuit begins and ends at the same vertex and covers every edge only once.

3. Refer to figure 5. Is the graph connected? Explain.

4. Can you trace an Euler circuit in figure 5?

5. Based on your answers above, what property can you conjecture about a graph that has an Euler circuit?

6. In your own words, what is another condition that causes a graph to lack an Euler circuit?

Extension:

Based on your work in activities 2 and 3, what are the two conditions you would examine to determine whether a graph has an Euler circuit. Summarize your findings below.

Eulerizing a Circuit

Name: ____

Activity #4 – Eulerization of a Circuit:

1. Draw a graph that is connected, but has at least one vertex with an odd valence. Keep your graph relatively simple!

2. Based on the fact that a graph with an Euler circuit must have even valences at all vertices and be connected, how could you change your graph above to meet this criteria? Explain below and draw your changes in the figure above.

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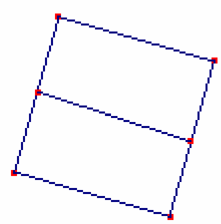
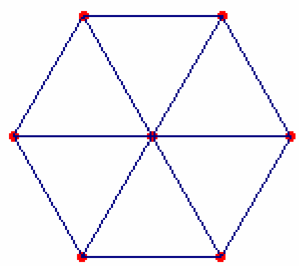
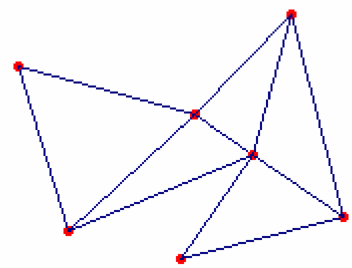
3. Draw a graph that is not connected, but has even valences at all vertices. Keep your graph relatively simple!

4. Based on the fact that a graph with an Euler circuit must be connected and have even valences at all vertices, how could you change your graph

above to meet this criteria? Explain below and draw your changes in the figure above.

** This method is called the ***Eulerization of a circuit***!

5. Refer to the three graphs below. Can you Eulerize these circuits? Draw your changes on the figures below.

		
Graph A	Graph B	Graph C

6. Can you generalize a method for Eulerizing a given graph? Explain your reasoning.

7. In your own words, how would you describe the process of Eulerizing a circuit?

8. Can you think of any practical applications where Eulerizing a circuit might be helpful?

Extension:
If the mayor of a city is designing a route for snowplows, why would the process of Eulerization be particularly useful? (Hint: start by drawing a graph to represent several city blocks)

Extension:
Figure 1 is a vertex-edge graph. We cannot find an Euler Circuit in this graph because there is at least one vertex with odd valence.

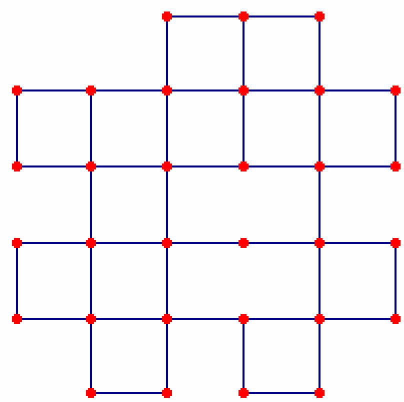


Figure 1

1. Below are 3 different Eulerizations of Figure 1. If given a choice, which one would you choose if, for example, this vertex-edge graph represented roads in a city? Why would you choose this one?

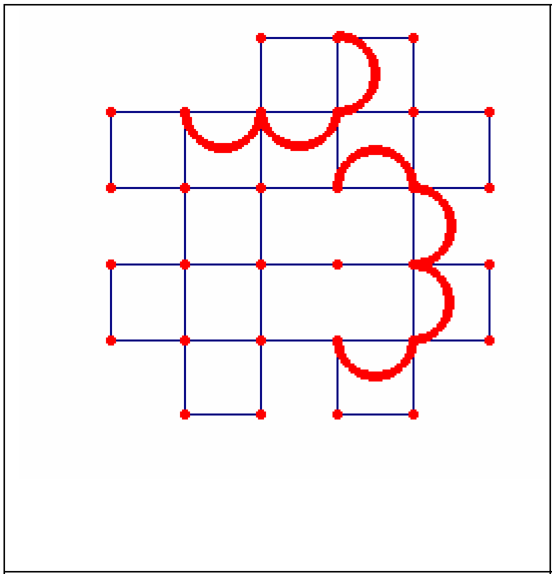


Figure 2

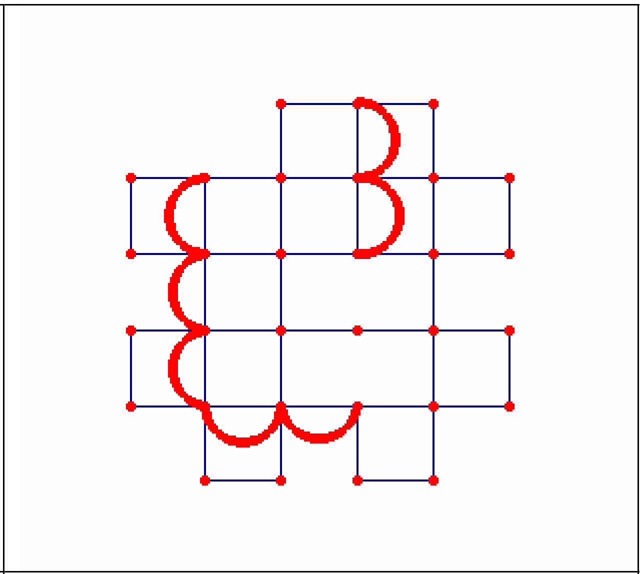


Figure 3

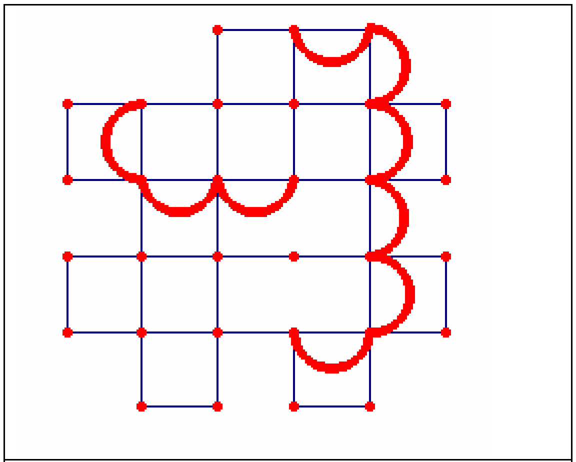


Figure 4

2. Figure 5 is another vertex-edge graph. What is the minimal number of edges that you have to add to get an Eulerization? Draw a few Eulerizations to convince yourself that you have the minimal number.

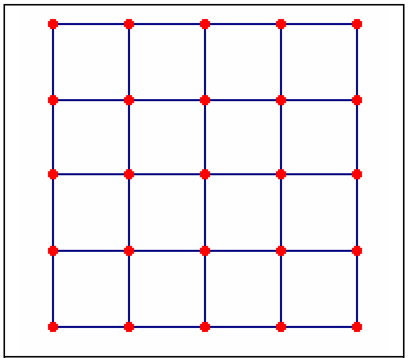


Figure 5