

Fair Division

1. Introduction to Fair Division
2. The Divider Chooser Method
3. The Lone Divider Method
4. The Method of Sealed Bids

- I. Intro to fair division: <https://www.youtube.com/watch?v=e7WO1lwW36s&t=6s>
 - a. Define a fair division method
 - b. Define fair share
 - c. Given a situation, determine a person's fair share.

- II. The Divider Chooser Method

Two Players:

- The divider cuts the item into two pieces that are, in the players opinion, equal in value.
- The chooser selects either of the two pieces.
- Divider receives remaining piece.
- Only gives a fair division and not necessarily the best division.

Ex.1: Mr. Miller and Mr. Switzer want to split a bag of fun sized candy and decide to use the divider chooser method. The bag contains 100 Snickers, 100 Milky Way and 100 Reese's. Mr. Miller values each at \$3, \$2 and \$1.

What is the value of each individual piece of candy?

S _____

MW _____

R _____

Mr. Switzer is the divider and in one-half puts 45 Snickers, 45 Milky Way and 65 Reese's. What is the value of this for Mr. Miller and is this a fair share?

Ex.2: Mr. Miller and Mr. Switzer want to split a bag of fun sized candy and decide to use the divider chooser method. The bag contains 100 Snickers, 100 Milky Way and 100 Reese's. Mr. Switzer values each at \$2, \$1 and \$3.

What is the value of each individual piece of candy?

S _____

MW _____

R _____

Mr. Miller is the divider and in one-half puts 40 Snickers, 45 Milky Way, and 65 Reese's.

What is the value of this half for Mr. Switzer and does he find this half to be fair?

Ex.3: Matt and Nick are trying to fairly split a 32" sub that costs \$21. Matt likes ham twice as much as turkey. He likes turkey and roast beef the same. Nick likes roast beef twice as much as ham. He likes turkey and ham the same. How would Matt cut the sandwich fairly?

III. The Lone Divider Method

- Works for any number of players.
- "N" number of players.
- One person randomly assigned the divider and the rest are choosers.

The Process

- Divider divides item into "N" pieces which we will call $S_1, S_2, S_3, \dots, S_n$.
 - Each chooser will separately list which pieces they consider to be a fair share. This is called their declaration or bid.
 - Examine lists: two possible outcomes
 - If it is possible to give each party a piece they declared, then do so. Divider gets remaining.
- Or
- If two or more parties want the same pieces and no others (standoff), then give a non-contested piece to the divider. Combine remaining pieces and repeat the entire procedure with the remaining parties. If there are only two parties left, use the divider chooser method.

Ex.1: Three investors are dividing land worth \$300,000. Each investor value in thousands for each of the three pieces of land is shown below.

	Piece 1	Piece 2	Piece 3
Juan	80	100	120
Linda	100	100	100
Martin	60	90	150

What is each investor's fair share?

Who is the divider? How do you know?

Declarations:

Final Allocations:

Juan:

Linda:

Martin:

Ex.2: Four people are dividing a piece of land. The value of each plot are shown below.

	Piece 1	Piece 2	Piece 3	Piece 4
Samantha	21%	18%	34%	27%
Kelly	30%	21%	23%	26%
Catherine	22%	21%	42%	15%
Renee	25%	25%	25%	25%

What is each person's fair share? How do you know?

Who is the divider? How do you know?

Declarations:

Which piece does each person receive?

Samantha:

Kelly:

Catherine:

Renee:

Ex. 3:

Six friends agree to divide a \$18 pizza fairly using the **Lone Divider Method**. The table shows how each player values each of the six slices that have been cut by the divider. Assume that all of the friends play honestly.

	s_1	s_2	s_3	s_4	s_5	s_6
Tuco	\$1	\$2	\$1	\$1	\$1	\$13
Hector	\$3	\$3	\$3	\$3	\$3	\$3
Leonel	\$8	\$2	\$2	\$2	\$2	\$2
Marco	\$5	\$2	\$3	\$2	\$2	\$4
Tortuga	\$2	\$2	\$8	\$1	\$3	\$2
Don Eliado	\$10	\$1	\$0	\$4	\$0	\$3

- (a) How much is a fair share worth?
- (b) Which player was the Divider?
- (c) Give the bid list (declaration) for each player.

player	Tuco	Hector	Leonel	Marco	Tortuga	Don Eliado
bid list						

- (d) Give a possible Fair Division of the pizza, if one exists.

IV. The Method of Sealed Bids

- Discrete fair division
- Allows the division of items that cannot be split into smaller pieces
 - a. House
 - b. Car
 - c. Boat
- Requires all parties to have a large amount of cash at their disposal to balance out the difference in item value.
- This method can be used for things such as divorce settlement, death (items left in a will) or dissolving of business assets.

The Process

1. Compile a list of items to be divided.
2. Each party involved lists, in secret, a dollar amount they value each item to be worth. This is their sealed bid.
3. Bids are collected. For each party, the value of all the items is totaled, and divided by the number of parties. This defines their fair share.
4. Each item is awarded to the highest bidder.
5. For each party, the value of all items received is totaled. If the value is more than that party's fair share, they pay the difference into a holding pile. If the value is less than that party's fair share, they receive the difference from the holding pile. This ends the initial allocation.
6. In most cases, there will be a surplus, or leftover money, in the holding pile. The surplus is divided evenly between all the players. This produces the final allocation.

Ex.1: A grandmother leaves all of her assets to charity except three items. She leaves her house, car and diamond ring to her three grandchildren. Each grandchild honestly and independently place the following bids in the thousands of dollars. Divide up these assets using a sealed bid method.

	P1	P2	P3
House	150	170	125
Car	6	8	10
Ring	3	5	3
Total			
Fair Share			

Ex. 2: Four heirs (A, B, C, D) must fairly divide an estate consisting of two items- a desk and a vanity- using the method of sealed bids. The players' bids (in dollars) are shown below.

	A	B	C	D
Desk	300	280	260	200
Vanity	180	120	140	100
Total				
Fair Share				

What is the original fair share of A worth?

What happens with Player A in the initial allocation?

After the process, what is the final outcome for all the players?

Ex. 3: Samantha and Renee were college roommates for four years. During this time they shared the expense of purchasing several items for the apartment. After graduation Samantha accepted a job in another city and now they find themselves needing to divide their shared assets. Give the final allocation given the valuations shown using the Sealed Bids method.

	Samantha	Renee
Couch	\$150	\$100
TV	\$200	\$250
PS4	\$250	\$150
Surround Sound	\$50	\$100
Total		
Fair Share		

What is Samantha's fair share worth? Renee's?

What is the final allocation for both girls?