Honors Discrete Matrix Review Worksheet

Name Key

Name Key

For questions 1 - 4 refer to the following matrices.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 1 & 6 & 8 \\ 4 & 0 & -3 & 2 \end{bmatrix}$$

1. What are the dimensions of A?

3×2

2. What are the dimensions of B?

2×4

3. What is A_{22} ?

$A_{22} = 0$

4. What is B_{21} ?

$B_{21} = 4$

For questions 5 - 14, refer to the following matrices.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 1 & 6 & 8 \\ 4 & 0 & -3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 4 & -2 \\ 2 & 1 & 6 \\ 0 & 3 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} \quad G = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 0 & -4 \end{bmatrix} \quad H = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -3 \\ 0 & -4 & 0 \end{bmatrix}$$

Find the following.

5. 3A $3\begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 6 & 0 \\ -3 & 12 \end{bmatrix}$

6. ½ D $\frac{1}{2}\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 1 \\ -\frac{1}{2} & 2 \end{bmatrix}$

7. F - 2D $\begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 4 \\ -2 & 8 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ 2 & -10 \end{bmatrix}$

8. D + 3F $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} + 3\begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 24 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 6 & 26 \\ -1 & -2 \end{bmatrix}$

9. DB $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 & 6 & 8 \\ 4 & 0 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 12 & 28 \\ 18 & -1 & -18 & 0 \end{bmatrix}$

10. EH $\begin{bmatrix} 1 & 4 & -2 \\ 2 & 1 & 6 \\ 0 & 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -3 \\ 0 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 7 & -11 \\ 2 & -26 & -1 \\ 6 & 0 & -9 \end{bmatrix}$

11. F^2

$$\begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix}^2 = \begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & 4 \end{bmatrix}$$

12. AF

$$\begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 22 \\ 2 & 16 \\ -1 & -16 \end{bmatrix}$$

13. $|D|$ ← determinant

$$\begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix} = 12 - (-2) = 14$$

14. $|E|$ ← determinant

$$\begin{vmatrix} 1 & 4 & -2 \\ 2 & 1 & 6 \\ 0 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 4 \\ 2 & 1 \\ 0 & 3 \end{matrix}$$

$(0+0-12) - (0+18+0)$

$-12-18$

-30

For questions 15 - 20, refer to the following matrices.

$$A = \begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 2 \\ 15 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$$

15. Find the inverse of A.

$\begin{vmatrix} 3 & 1 \\ -4 & 1 \end{vmatrix} = 3+4 = 7$

$\frac{1}{7}\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ \frac{4}{7} & \frac{3}{7} \end{bmatrix}$

16. Find the inverse of B.

$\begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} = 6 - 0 = 6$

$\frac{1}{6}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$

Find the missing matrix.

17. $EA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$E\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$E = A^{-1} \cdot B$

$E = \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ \frac{4}{7} & \frac{3}{7} \end{bmatrix}$

18. $AF = \begin{bmatrix} 11 & 2 \\ -17 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}F = \begin{bmatrix} 11 & 2 \\ -17 & 2 \end{bmatrix}$

$F = A^{-1} \cdot B$

$F = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

19. $GD = \begin{bmatrix} 1 & 3 \\ 9 & 2 \\ 2 & 1 \end{bmatrix}$

$G \cdot \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & 2 \\ 2 & 1 \end{bmatrix}$

$G = A^{-1} \cdot B$

$2\times2 \quad 3\times2$

can do

$G = \emptyset$

20. $BL = \begin{bmatrix} 2 \\ -13 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} \cdot L = \begin{bmatrix} 2 \\ -13 \end{bmatrix}$

$L = A^{-1} \cdot B$

$L = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

Solve Question 21 by using Cramer's Rule and 22 by Inverse Matrices.

21. $2x - 3y = 32$
$x + 4y = -20$

$\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} = 8 + 3 = 11$

$X = \dfrac{\begin{vmatrix} 32 & -3 \\ -20 & 4 \end{vmatrix}}{11}$ $\quad y = \dfrac{\begin{vmatrix} 2 & 32 \\ 1 & -20 \end{vmatrix}}{11}$

$X = \dfrac{128 - 60}{11}$ $\quad y = \dfrac{-40 - 32}{11}$

$X = \dfrac{68}{11}$ $\quad y = \dfrac{-72}{11}$

$\left(\dfrac{68}{11}, \dfrac{-72}{11}\right)$

22. $2x + y - z = 15$
$4x - 3y + 7z = -11$
$x + y + z = 2$

$\begin{bmatrix} 2 & 1 & -1 \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ -11 \\ 2 \end{bmatrix}$

A $\qquad$ B

$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix}$

For questions 23 - 24, solve each system of equations by using the inverse matrix method.

23. $x + 4y = -19$
$-3x + 2y = -13$

$\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -19 \\ -13 \end{bmatrix}$

A $\qquad$ B

$A \begin{bmatrix} x \\ y \end{bmatrix} = B$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

24. $x + 4y = -2$
$-3x + 2y = 6$

$\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

A $\qquad$ B

$A \begin{bmatrix} x \\ y \end{bmatrix} = B$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

25. An advertisement from the back page of the Denton Chronicle:

$1300

$1400

$1600

Sofa and love seat

Sofa and two chairs

Sofa, love seat, one chair

How much does each piece of furniture cost individually?(Create a system and use Inverse Matrices to solve)

$S + \ell = 1300$
$S + 2C = 1400$
$S + \ell + C = 1600$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} S \\ \ell \\ C \end{bmatrix} = \begin{bmatrix} 1300 \\ 1400 \\ 1600 \end{bmatrix}$$

A B

$$A\begin{bmatrix} S \\ \ell \\ C \end{bmatrix} = B \qquad \begin{bmatrix} S \\ \ell \\ C \end{bmatrix} = A^{-1} \cdot B \qquad \begin{bmatrix} S \\ \ell \\ C \end{bmatrix} = \begin{bmatrix} 800 \\ 500 \\ 300 \end{bmatrix}$$

26. The table below shows the percent of comedies, drama, and action videos available at a video store. Assume that the store has a collection of 3,405 general videos to be rented, 1,070 children's videos to be rented, and 1,225 videos for sale. Write and solve a system of equations to find out how many comedies, dramas, and action movies are at the store.

Store Section	Comedy	Drama	Action
General rental	55%	65%	60%
Children's rental	25%	10%	20%
Videos for sale	20%	25%	20%

$$\begin{bmatrix} .55 & .65 & .60 \\ .25 & .10 & .20 \\ .20 & .25 & .20 \end{bmatrix}\begin{bmatrix} c \\ d \\ a \end{bmatrix} = \begin{bmatrix} 3405 \\ 1070 \\ 1225 \end{bmatrix}$$

A B

$$A\begin{bmatrix} c \\ d \\ a \end{bmatrix} = B \qquad \begin{bmatrix} c \\ d \\ a \end{bmatrix} = A^{-1} \cdot B \qquad \begin{bmatrix} c \\ d \\ a \end{bmatrix} = \begin{bmatrix} 2000 \\ 1700 \\ 2000 \end{bmatrix}$$