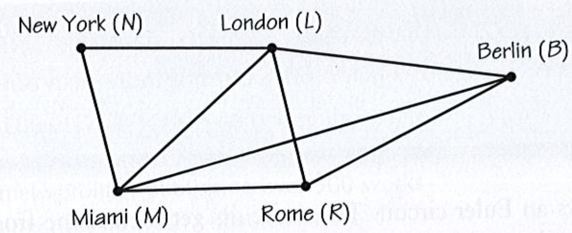


FIGURE 1.2 The edges of the graph show nonstop routes that an airline might offer.

the edges *LR* and *MB* do not cross, or redraw the diagram so as to avoid a crossing in this case (but not in all graphs we might wish to draw). We will be working often in situations where graphs can be drawn without accidental crossings and we will try to avoid such crossings when it is convenient to do so.

Returning to the case of parking control in Figure 1.1, we can use a graph to represent the whole territory to be patrolled: Think of each street intersection as a vertex and each sidewalk that contains meters as an edge, as in Figure 1.3. Notice in Figure 1.3b that the width of the street separating the blocks is not explicitly represented; it has been shrunk to nothing. In effect, we are simplifying our problem by ignoring any distance traveled in crossing streets.

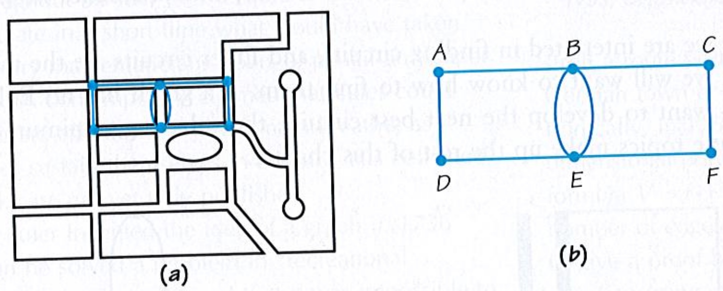


FIGURE 1.3 (a) A graph superimposed upon a street map. The edges show which sidewalks have parking meters. (b) The same graph enlarged.

The numbered sequence of edges in Figure 1.4a shows one circuit that covers all the meters (note that it is a circuit because its path returns to its starting point). However, one edge is traversed three times. Figure 1.4b shows another solution that is better because its circuit covers every edge (sidewalk) exactly once. In Figure 1.4b, no edge is covered more than once, or *deadheaded* (a term borrowed from shipping, which means making a return trip without a load).

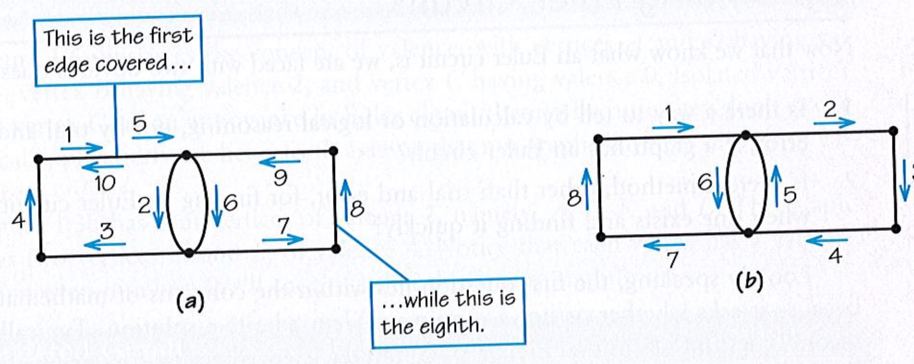


FIGURE 1.4 (a) A circuit and (b) an Euler circuit.

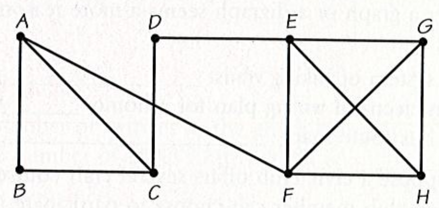
CHAPTER 1 EXERCISES

■ Challenge ◆ Discussion

1.1 Euler Circuits
1.2 Finding Euler Circuits

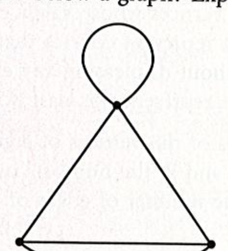
1. In the graph below, the vertices represent houses and two vertices are joined by an edge if it is possible to drive between the two houses in under 10 minutes.

(a) How many vertices does the graph have?
(b) How many edges does the graph have?
(c) What are the valences of the vertices in this graph?
(d) Based on the information given by the graph below, for which houses, if any, is it possible to drive to all the other houses in less than 20 minutes?
(e) Based on the graph below, from house *B* which houses require a trip of longer than 20 minutes?

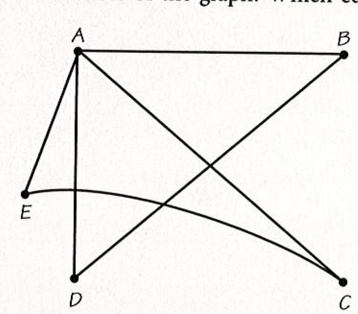


2. (a) Redraw the graph in Figure 1.2 to obtain a graph which has the same information where the edges only meet other edges at vertices.
(b) List all the routes that start on the U.S. side of the Atlantic Ocean and cross the ocean once and immediately.

3. (a) Is the figure below a graph? Explain your answer.

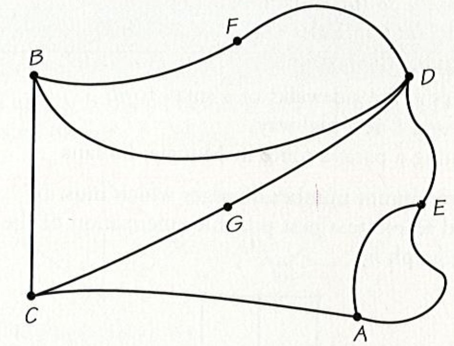


(b) The graph below has edges that "cross" at points that are not vertices of the graph. Which edges are these?



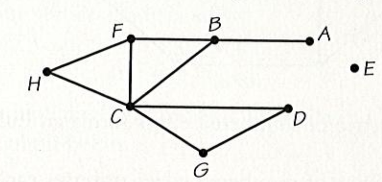
(c) How many vertices and edges are there in the prededing graph?

4. The graph below shows the stores and roads connecting them in a small shopping mall.

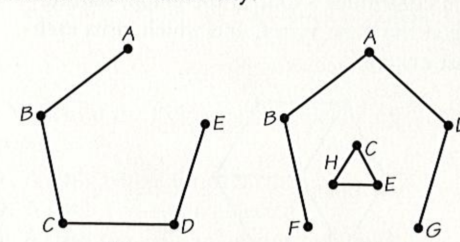


(a) How many stores does the mall have?
(b) How many roads connect up the stores in the mall?
(c) Write down a path from *C* to *F*.
(d) Write down a path from *E* to *B*.

5. In the graph below, the vertices represent cities and the edges represent roads connecting them. What are the valences of the vertices in this graph? (Keep in mind that *E* is part of the graph.) What might the valence of city *E* be showing about the geography?



6. In the two graphs below, the vertices represent cities and the edges represent roads connecting them. In which graphs could a person located in city *A* choose any other city and then find a sequence of roads to get from *A* to that other city?



(a) (b)

7. Refer to the figure in Exercise 4.
(a) Write down a circuit that includes the vertices *C* and *D* but does not start or end at either of these vertices.
(b) If two paths are considered different if they use different edges, write down:
  (i) two different paths from *B* to *D*.
  (ii) three different paths from *C* to *F*.
  (iii) a circuit that has four edges.

8. Jack and Jill are located in Miami and want to fly to Berlin (see Figure 1.2).
(a) Find three paths for them to carry out this trip.
(b) What is the largest number of paths that can be used to carry out this trip that do not repeat a vertex (city)?
(c) Explain why it is reasonable not to want to repeat a vertex in this situation.

9. (a) How many vertices and edges does the graph in Figure 1.6 have?
(b) How many vertices and edges does the graph in Figure 1.7 have?
(c) How many vertices and edges does the graph in Figure 1.8a have?

10. (a) Add up the numbers you get for the valences of the vertices in Figure 1.6.
(b) Add up the numbers you get for the valences of the vertices in Figure 1.7.
(c) Add up the numbers you get for the valences of the vertices in Figure 1.8a.
(d) Describe the pattern you see in the answers you got for parts (a) through (c).
(e) Show that the pattern describes a fact that is true for any graph. (*Hint:* How many endpoints does an edge have?)

11. In the graph in Figure 1.8a, find the smallest possible number of edges you could remove that would disconnect the graph.

12. In the graphs in Figure 1.17, find the smallest possible number of edges you could remove that would disconnect the graph.

13. Draw a graph with eight vertices that is connected where
(a) each vertex has valence 3.
(b) each vertex has valence 4.
(c) Do all graphs with eight vertices having valence 2 have the same number of edges?

14. Is it possible that a street network gives rise to a disconnected graph? If so, draw such a network of blocks and streets and parking meters (in the style of Figure 1.12a). Then draw the disconnected graph it gives rise to.

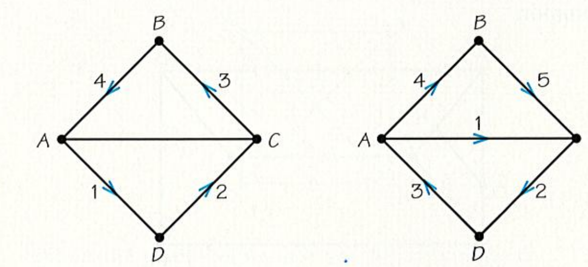
15. (a) Draw a connected graph with six vertices, all of whose vertices have valence 2.
(b) Draw a disconnected graph with 6 vertices, all of whose vertices have valence 2.

16. (a) Draw a graph where every vertex has valence of at least 3 but where removing a single edge disconnects the graph.
(b) In what urban settings might a road network be represented by a graph that has an edge whose removal would disconnect the graph?

17. (a) Find a graph where the valences of the seven vertices of the graph are 1, 2, 2, 3, 3, 3, 4.
(b) Find another graph with the same valences as above that is "different" from the one you found for part (a).

18. For some services provided along streets, it may matter whether the roads are one-way or two-way. Give some examples where the street directions do and do not matter for our graph model analysis.

◆ **19.** A postal worker is supposed to deliver mail on all streets represented by edges in the graph below by traversing each edge exactly once. The first day the worker traverses the numbered edges in the order shown in (a), but the supervisor is not satisfied–why? The second day the worker follows the path indicated in (b), and the worker is unhappy–why? Is the original job description realistic? Why?



(a) (b)

20. For the street network in Exercise 19, draw the graph that would be useful for routing a snowplow. Assume that all streets are two-way, one lane in each direction, and that you need to pass down each lane separately.

21. Find an efficient route for the snowplow to follow in the graph you drew in Exercise 20.

22. (a) Give examples of services that could be performed by a vehicle that moved in the direction of traffic down either lane of a two-way street.
(b) Give examples of services that would probably require a vehicle to travel down each of the lanes of a two-way street (in the direction of traffic for that lane) to perform the service.