

◆ 19. In a few sentences, explain why plurality voting satisfies

- (a) the Pareto condition.
- (b) monotonicity.

◆ 20. In a few sentences, explain why the Borda count satisfies

- (a) the Pareto condition.
- (b) monotonicity.

◆ 21. In a few sentences, explain why sequential pairwise voting satisfies

- (a) the Condorcet winner criterion.
- (b) monotonicity.

◆ 22. In a few sentences, explain why the Hare system satisfies the Pareto condition.

◆ 23. In a few sentences, explain why the plurality runoff method satisfies the Pareto condition.

■ 24. Use the following ballots to show that the plurality runoff method does not satisfy the Condorcet winner criterion:

Rank	Number of Voters (5)		
	2	2	1
First	A	B	C
Second	C	C	B
Third	B	A	A

■ 25. Use the following ballots to show that the plurality runoff method does not satisfy monotonicity:

Rank	Number of Voters (13)				
	4	3	3	2	1
First	A	B	C	D	E
Second	B	A	A	B	D
Third	C	C	B	C	C
Fourth	D	D	D	A	B
Fifth	E	E	E	E	A

26. Consider the following two elections among candidates A, B, and C:

Rank	Number of Voters (4)			
	1	1	1	1
First	A	A	B	C
Second	B	B	C	B
Third	C	C	A	A

Rank	Number of Voters (4)			
	1	1	1	1
First	A	A	B	B
Second	B	B	C	C
Third	C	C	A	A

(a) Use these two elections to show that plurality voting does not satisfy independence of irrelevant alternatives.

(b) Use these two elections to show that the Hare system does not satisfy independence of irrelevant alternative.

■ 27. Construct ballots for the alternatives A, B, and C to show that the Borda count does not satisfy the Condorcet winner criterion.

28. Show that the nonmonotonicity of the Hare system can also be demonstrated by the following 17-voter, 4-alternative election. (In a number of recent books, this example is used to show the nonmonotonicity of the Hare system. The 13-voter, 3-alternative example given in the text was pointed out to us by Matt Gendron, an undergraduate at Union College.)

Rank	Number of Voters (17)			
	7	5	4	1
First	A	C	B	D
Second	D	A	C	B
Third	B	B	D	A
Fourth	C	D	A	C

29. The following example illustrates how badly the Hare system can fail to satisfy monotonicity. Consider the following sequence of preference lists:

Rank	Number of Voters (21)			
	7	6	5	3
First	A	B	C	D
Second	B	A	B	C
Third	C	C	A	B
Fourth	D	D	D	A

(a) Show that A is the unique winner if the Hare system is used.

(b) Find the winner using the Hare system in the new election wherein the three voters on the right all move A from last place on their preference lists to first place on their preference lists.

◆ 30. In a few sentences, explain why, with an odd number of voters,

(a) sequential pairwise voting always yields a unique winner.

(b) we can never have exactly two winners with the Hare system.

◆ 31. In a few sentences, explain why the plurality runoff method can never elect a candidate ranked last on a majority of ballots, assuming there are no ties for first or second place in the voting.

32. Produce ballots showing that plurality voting can, in fact, elect a candidate ranked last on a majority of the ballots.

33. Suppose there are three voters and three alternatives: A, B, and C.

(a) If each alternative has exactly one first-place vote, what is the election outcome if the Hare procedure is used? What if plurality runoff is used?

(b) If an alternative has two or more first-place votes, what is the election outcome if the Hare procedure is used? What if plurality runoff is used?

(c) Can the Hare procedure and plurality runoff yield different election outcomes when there are three voters and three alternatives? Explain your answer in one sentence.

9.3 Insurmountable Difficulties: Arrow's Impossibility Theorem

■ 34. Complete the proof of the version of Arrow's theorem from the text by showing that neither B nor C can be a winner in the situation described. (Your argument will be almost word for word the same as the proofs in the text.)

9.4 A Better Approach? Approval Voting

35. Ten board members vote by approval voting on eight candidates for new positions on their board as indicated in the following table. An X indicates an approval vote. For example, Voter 1, in the first column, approves of candidates A, D, E, F, and G, and disapproves of B, C, and H.

Candidate	Voters									
	1	2	3	4	5	6	7	8	9	10
A	X	X	X			X	X	X		X
B		X	X	X	X	X	X	X	X	
C			X					X		
D	X	X	X	X	X		X	X	X	X
E	X	X	X		X		X		X	
F	X		X	X	X	X	X	X		X
G	X	X	X	X	X			X		
H		X		X		X		X		X

(a) Which candidate is chosen for the board if just one of them is to be elected?

(b) Which candidates are chosen if the top four are selected?

(c) Which candidates are elected if 80% approval is necessary and at most four are elected?

(d) Which candidates are elected if 60% approval is necessary and at most four are elected?

36. The 45 members of a school's football team vote on three nominees, A, B, and C, by approval voting for the award of "most improved player" as indicated in the following table. An X indicates an approval vote.

Nominee	Number of Voters (45)					
	7	8	9	9	6	3
A	X			X	X	X
B		X		X		X
C			X		X	X

(a) Which nominee is selected for the award?

(b) Which nominee gets announced as runner-up for the award?

(c) Note that two of the players "abstained," that is, approved of none of the nominees. Note also that one person approved of all three of the nominees. What would be the difference in the outcome if one were to "abstain" or "approve of everyone"?