Solutions

Skills Check:

1. a
2. 7; 8
3. a
4. 3
5. c
6. *B*; *E*
7. b
8. 6
9. c
10. 6
11. a
12. 4
13. b
14. 12
15. a
16. 3
17. b
18. digraph; graph; digraph
19. b
20. 8; 13

Cooperative Learning:
Euler Circuits:

a. Diagrams (2), (3), (4), (5), (7), and (8) can be drawn without lifting the pencil from the paper.

b. In (4) and (5) you can return to the starting point.

c. In part a, there are at most 2 vertices with odd valences, while in part b there are no vertices with odd valences.

Eulerization:

(1) Duplicate 1 edge.

(2) Duplicate 2 edges.

(3) Duplicate 4 edges.

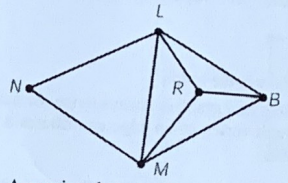
(4) Duplicate 9 edges.

If there are 4 vertices with odd valences, then there will be a *minimum* of two duplications. However, this minimum can't always be achieved, as we see from numbers (3) and (4).

Exercises:

1. **(a)** There are 8 vertices.
   (b) There are 12 edges.
   (c) *A* has valence 3, *B* has valence 2, *C* has valence 3, *D* has valence 2, *E* has valence 4, *F* has valence 4, *G* has valence 3, and *H* has valence 3.
   (d) This is the same as asking if given a vertex, can you reach any other vertex involving at most 2 edges. *A* can reach *B*, *C*, and *F* via one edge. *A* can reach *D*, *E*, *G*, and *H* via two edges. *D* and *F* can similarly reach all other houses in under 20 minutes.
   (e) *B* can reach *A* and *C* in under 10 minutes. *B* can also reach *D* and *F* in under 20 minutes. However, to reach *E*, *G* and *H*, more than 20 minutes is required.

2. **(a)**



   (b) Assuming that no cities are revisited and that one goes directly from an American city to a European one the routes are:
   NL, *NLR*, *NLB*, *NLBR*, *NLRB*,
   ML, *MR*, *MB*, *MLR*, *MLB*, *MRL*, *MRB*, *MBL*, *MBR*, *MLRB*, *MLBR*, *MRLB*, *MRBL*, *MBLR*, *MBRL*

3. **(a)** This diagram fails to be a graph because a line segment joins a single vertex to itself. The definition being used does not allow this.
   (b) The edge *EC* crosses edges *AD* and *BD* at points which are not vertices; edge *AC* crosses *BD* at a point that is not a vertex.
   (c) This graph has 5 vertices and 6 edges.

4. **(a)** 7 stores
   (b) 10 roads
   (c) *CBF* is a path.
   (d) *EDFB* is a path.

5. *E* has valence 0; *A* has valence 1; *H*, *D*, and *G* have valence 2; *B* and *F* have valence 3; *C* has valence 5. *E* is "isolated." *E* might have valence 0 because it is on an island with no road access.

6. **(a)** Yes.
   (b) No. There is no way to get from *A* to *C*, *E*, or *H*.

7. **(a)** *BCGDFB*
   (b) **(i)** *BD*; *BFD*
   (ii) *CBF*; *CGDF*; *CGDBF*
   (c) *GDBCG*

8. **(a)** There are 5 paths to choose from if you assume that no cities are revisited and that one goes directly from an American city to a European city. These are *MLB*, *MLRB*, *MRB*, *MRLB*, and *MB*. If you assume that you can travel between Miami and New York prior to going to Europe then you would consider the paths *MNLB* and *MNLRB*. If a path is allowed to revisiting a city, then there are infinitely many paths.
   (b) There are 5 paths if domestic travel is not considered and 7 paths to choose from if domestic travel is allowed.
   (c) In such a route, time and probably cost would be greater if you repeat a city prior to getting to the destination.

9. **(a)** 4 vertices; 4 edges.
   (b) 7 vertices; 6 edges.
   (c) 10 vertices; 14 edges.