10. **(a)** $2+3+3+0=8$

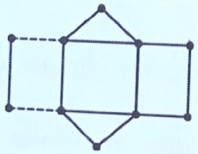
(b) $2+2+2+2+2+1+1=12$

(c) 28

(d) The number we obtain is twice the number of edges in the graph.

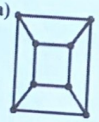
(e) The fact that the sum of the valences of the vertices of a graph is always twice the number of edges in the graph follows from noticing that each vertex of an edge contributes a total of two to the sum because the edge has two endpoints.

11. Remove the edges dotted in the figure below and the remaining graph will be disconnected.

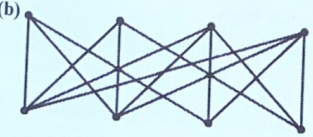


12. In any of these graphs, two edges can be removed and the graph will become disconnected. One of the disconnected pieces will be a single vertex.

13. **(a)**

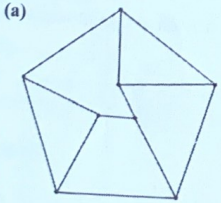


(b)

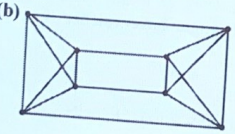


Drawings can vary. Some other graphs could be as follows.

(a)

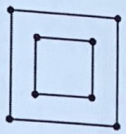


(b)



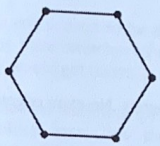
(c) Yes. The sum of the valences of a graph with 8 vertices each of valence 2 is 16. Thus, all such graphs have 8 edges.

14. Yes, a disconnected graph can arise. One, possible example is shown below:

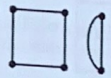


which gives rise to the disconnected graph:

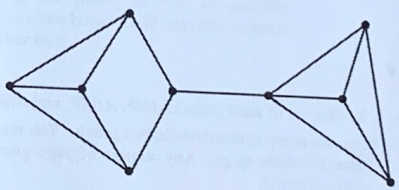
15. **(a)**



(b)

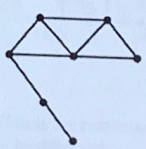


16. **(a)**

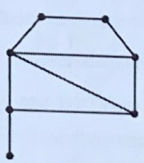


(b) The edge might represent a bridge or tunnel. Recently, when a bridge collapsed because it was hit by a barge, there was a major disruption to the communities near the bridge on opposite sides of the river.

17. **(a)**



(b)



18. The street direction will matter for a problem involving how long it will take to get between two street intersections and for routing a street sweeper that follows traffic rules. The street direction may not matter for an inspector checking "manholes" located in the middle of streets or a service that involves walking along either side of the street such as inspecting sidewalks.

19. The supervisor is not satisfied because all of the edges are not traveled upon by the postal worker. The worker is unhappy because the end of the worker's route wasn't the same point as where the worker began. The original job description is unrealistic because there is no Euler circuit in the graph.