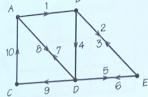
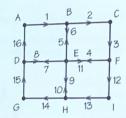
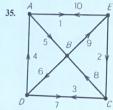
- 34. The curved edges on the first graph become double-traversals on the straight edges of the second graph.



## ABEBDEDADCA



## ABEBDEDADCA



## **AECDABDCBEA**

36. A minimum of three edges must be added: one edge along the horizontal segment in the first parallelogram and a segment along two opposite edges of the second parallelogram.







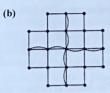


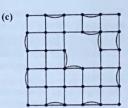




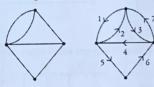








- 39. (a) There are four 3-valent vertices. By properly removing two edges adjacent to these four vertices, (edge between left two 3-valent vertices and edge between right two 3-valent vertices) one can make the graph even-valent.
  - (b) Yes, because the resulting graph is connected and even-valent.
  - (c) It is possible to remove two edges and have the resulting graph be even-valent.
  - (d) No, because the resulting graph is not connected, even though it is even-valent.
- 40. Represent each riverbank by a vertex and each island by a vertex. Represent each bridge by an edge. This produces the graph on the left. After eulerization we produce the graph on the right. An Euler circuit is shown on this graph. After squeezing this circuit into the original graph, we have a circuit with one repeated edge.



- 41. There are many different circuits which will involve three reuses of edges. These are the edges which join up the six 3-valent vertices in pairs.
- 42. (a) (i) The number of edges added will always be the number of odd-valent vertices divided by
  - (ii) Since we must duplicate existing edges to find the best eulerization, the number of edges added will always be at least the number of odd-valent vertices divided by two, but often is a larger number.
  - (b) Yes.
- 43. The following graph satisfies the condition. You would eulerize the graph by duplicating each edge exactly once. The two end vertices are odd-valent.



44. The minimum length (34,000 feet) is obtained for any Euler circuit in the graph with edges duplicated as shown below. For minimizing total length it is better to repeat many shorter edges rather than a few long ones.



45. There are many circuits that achieve a length of 44,000 feet. The number of edges reused is eight because a shorter length tour can be found by repeating more shorter edges than fewer longer edges.