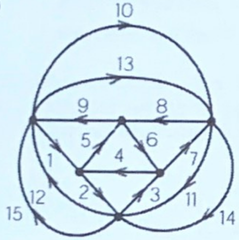
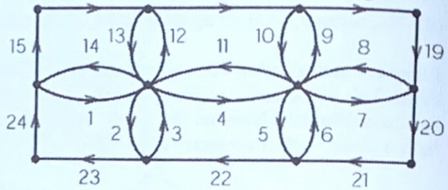
46. (a) The cheapest route has cost 49 and repeats edges BC, CD, and DF.
(b) Three edges.
(c) When there are different weights on the edges of a graph, the discussion about good eulerizations must be modified to take the size of the weights into account. It turns out there is an efficient, though complex, algorithm for finding minimum cost solutions to such problems.
(d) The weight might represent time. Two blocks of the same physical length can take different times to traverse due to construction or other factors.
(e) The weight might represent traversal time, traffic volume, number of potholes, number of stop signs, etc.

47. Both graphs (b) and (c) have Euler circuits. The valences of all of the vertices in (a) are odd, which makes it impossible to have an Euler circuit there.

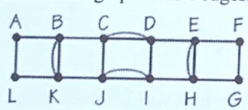
(b)



(c)



48. There are 5 different ways to eulerize this graph with 4 edges. One of them is shown below:

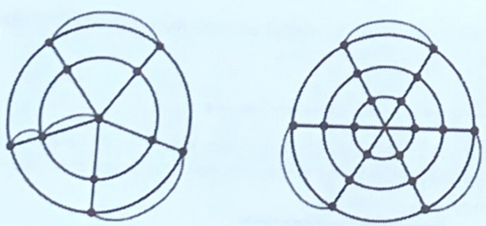


An Euler circuit in the original graph that repeats 4 edges is: *ABCDEHGFEHIJCDIJKBKLA*.

49. If the graph G is connected, the newly constructed graph will be even-valent and, thus, will have an Euler circuit. If G is not connected, the new graph will not have an Euler circuit because it, too, will not be connected.

50. A good eulerization duplicates the 5 "spokes" that go from the inner pentagon to the outer one. There are many Euler circuits in the eulerized graph.

51. (a)



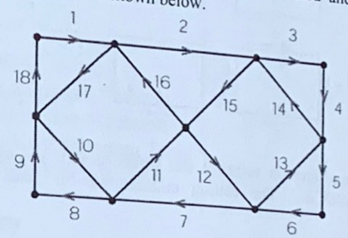
(b) The best eulerization for the four-circle, four-ray case adds two edges.
(c) Hint: Consider the cases where r is even and odd separately.

52. Pick any vertex and try to start an Euler circuit for the graph there. At some point the circuit traverses this special edge, crossing from the starting part of the graph to the other part. This special edge is the only connection between the parts, so we cannot return to the starting part and thus cannot have an Euler circuit. Since there is no Euler circuit, somewhere there must be a vertex with an odd valence.

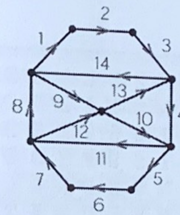
53. A graph with six vertices where each vertex is joined to every other vertex will have valence 5 for each vertex.

54. All graphs have Euler circuits because all graphs are connected and all vertices have even valences. Possible Euler circuits are shown below.

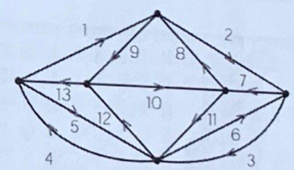
(a)



(b)



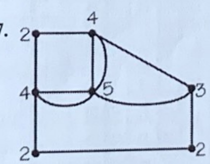
(c)



55. When you attach a new edge to an existing graph, it gets attached at two ends. At each of its ends, it makes the valence of the existing vertex go up by one. Thus the increase in the sum of the valences is two. Therefore, if the graph had an even sum of the valences before, it still does, and if its valence sum was odd before, it still is.

56. Dots without edges all have valence zero, and so the number of odd-valent vertices is zero, which is an even number. As edges are added, the number of odd-valent vertices will always increase by either 0 or 2. Thus, any graph has an even number of odd-valent vertices.

57.



The graph is connected.

58. When $r = 1$, a formula for the number of repeated edges is $(s-1)$. If r and s are odd, where $r = 2a+1$ and $s = 2b+1$ (a and b positive integers which are at least 1) then a formula for the number of repeated edges is $2(a+b)$. Similar formulas hold for the cases where both r and s are even or one of them is even and the other odd. The exact form of the formula depends on the way one expresses these situations.