Precalculus Sequences & Series Test Practice Name Key

Sequence Formulas:	$a_n = a_1 + d(n-1)$	$a_n = a_1 \cdot r^{n-1}$	
Series Formulas:	$S_n = \frac{n}{2}(a_1 + a_n)$	$S_n = \frac{a_1(1-r^n)}{1-r}$	$S_\infty = \frac{a_1}{1-r}$

Determine if the sequence is arithmetic or geometric. Find the common difference or the common ratio and write the equation for the nth term.

$a_n = -3(n-1) + 35$
$a_n = -3n + 3 + 35$

1) 35, 32, 29, 26, … A d or r = $d = -3$ a_n = $-3n + 38$

2) 6, 18, 54 … G d or r = $r = 3$ a_n = $6(3)^{n-1}$

Given the explicit formula for the sequence, find the first five terms and the named term in the problem.

3) $a_n = \frac{11}{8} + \frac{1}{2}(n-1)$ $\frac{11}{8}$, $\frac{15}{8}$, $\frac{19}{8}$, $\frac{23}{8}$, $\frac{27}{8}$ a_{23} = $\frac{99}{8}$

a_1, a_2, a_3, a_4, a_5

$a_n = \frac{11}{8} + \frac{1}{2}n - \frac{1}{2}$
$a_n = \frac{1}{2}n + \frac{7}{8}$

4) $a_n = 3^{n-1}$ 1, 3, 9, 27, 81 a_{18} = 129,000,000

a_1, a_2, a_3, a_4, a_5

Given the first term and the common difference of an *arithmetic* sequence find the first five terms and the explicit formula.

5) $a_1 = 28,\ d = 10$ 28, 38, 48, 58, 68

a_1, a_2, a_3, a_4, a_5

a_n = $10n + 18$

$a_n = 10(n-1) + 28$
$a_n = 10n - 10 + 28$
$a_n = 10n + 18$

Given the first term and the common ratio of a *geometric* sequence find the first five terms and the explicit formula.

6) $a_1 = 1,\ r = 2$ 1, 2, 4, 8, 16

a_1, a_2, a_3, a_4, a_5

a_n = $1(2)^{n-1}$

Find the first five terms using the given recursive formula

7) $a_1 = -1$

$a_{k+1} = (a_k)^2 + 9$

$a_{1+1} = (a_1)^2 + 9$ $a_{2+1} = (a_2)^2 + 9$

$\underline{-1}_{a_1}$, $\underline{10}_{a_2}$, $\underline{109}_{a_3}$, $\underline{11890}_{a_4}$, $\underline{141372109}_{a_5}$

$a_{3+1} = (a_3)^2 + 9$ $a_{4+1} = (a_4)^2 + 9$

Given two non-consecutive terms, find a_1, the common difference, and the explicit formula.

8) $a_9 = -5, a_{15} = 31$ $a_1 =$ -53 $d =$ 6

$31 = d(7-1) - 5$

$31 = 6d - 5$

$36 = 6d$

$d = 6$

$31 = 6(15-1) + a_1$

$31 = 6(14) + a_1$

$31 = 84 + a_1$

$\boxed{-53 = a_1}$

$a_n =$ $6n - 59$

$a_n = 6(n-1) - 53$

$a_n = 6n - 6 - 53$

$a_n = 6n - 59$

Given two non-consecutive terms, find a_1, the common ratio, and the explicit formula.

9) $a_4 = -192, a_7 = 12{,}288$ $a_1 =$ 3 $r =$ -4

$12{,}288 = -192r^{4-1}$

$-64 = r^3$

$\boxed{r = -4}$

$-192 = a_1(-4)^{4-1}$

$-192 = (-4)^3 \cdot a_1$

$-192 = -64a_1$

$\boxed{3 = a_1}$

$a_n =$ $3(-4)^{n-1}$

Expand and evaluate each series.

10) $\sum_{n=3}^{8} (n-3)^2$ $\boxed{= 55}$

$(3-3)^2 + (4-3)^2 + (5-3)^2 + (6-3)^2 + (7-3)^2 + (8-3)^2$

$0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \boxed{55}$

11) $\sum_{n=1}^{6} 2(.5)^{n-1}$ $= 2 + 1 + .5 + .25 + .125 + .0625$ $\boxed{= 3.9375}$

$2(.5)^{1-1} + 2(.5)^{2-1} + 2(.5)^{3-1} + 2(.5)^{4-1} + 2(.5)^{5-1} + 2(.5)^{6-1}$

~~2 + 1 + .5 + .25~~

Evaluate each arithmetic series using a sum formula. (Find the partial sum)

12) $\sum_{n=1}^{15} (4n)$

$S_{15} = \frac{15(4+60)}{2}$

$\boxed{S_{15} = 480}$

13) $20 + 27 + 34 + 41 \ldots, n = 16$

$a_1 = 20$

$a_{16} = 7(16-1) + 20$

$\boxed{a_{16} = 125}$

$S_{16} = \frac{16(20+125)}{2} = \boxed{1160}$

$\boxed{S_{16} = 1160}$

Evaluate each geometric series using a sum formula. (Find the partial sum)

$1\frac{1}{5}$

$\frac{6}{5}$

14) $\sum_{n=1}^{31} 2(1.2)^{n-1}$

$S_{31} = \frac{2(1-(1.2)^{31})}{1-1.2}$

$\boxed{S_{31} = 2838.515766}$

15) $-3 + -6 + -12 + -24 \ldots, n = 9$

$a_1 = -3$

$a_9 = (-3)(2)^{9-1}$

$\boxed{a_9 = -768}$

$S_9 = \frac{-3(1-2^9)}{1-2}$

$\boxed{S_9 = -1533}$

16) Decide which infinite geometric series has a sum.

a. ½ – 1 + 2 – 4 + … ~~Yes~~

b. 64 + 48 + 36 + 27 + … Yes

c. $\frac{1}{24}+\frac{1}{12}+\frac{1}{6}+\frac{1}{3}+\cdots$ Yes

d. 16 – 20 + 25 – 21.25 + … No

Evaluate the infinite geometric series, if possible.

$r = .375$

$a_1 = 800$

17) $800+300+\frac{225}{2}$

$S_n = \frac{800}{1-.375}$

$\boxed{S_n = 1280}$

18) $\sum_{k=1}^{\infty} \frac{8}{3}\left(\frac{1}{2}\right)^{k-1}$

$S_n = \frac{\frac{8}{3}}{1-\frac{1}{2}}$

$\boxed{S_n = \frac{16}{3}}$

Solve the given problems.

19) An auditorium contains 10 seats in the first row, 12 seats in the second, 14 in the third, and so on. How many seats are in the back row if there are 50 rows in the auditorium? How many total seats are in the auditorium?

$d = 2$

$a_1 = 10$

$a_{50} = 2(50-1)+10$

$a_{50} = 108$

$S_{50} = \frac{50(10+108)}{2}$

$\boxed{S_{50} = 2950}$

20) Ben started a job that paid \$40,000 a year. Each year after the first, his salary was increased by 4%. What was Ben's salary in his 8th year of employment? What is the total amount that Ben earned in eight years?

$a_8 = 40{,}000(1.04)^{8-1}$

$\boxed{a_8 \approx \$52{,}637}$

$S_8 = \frac{40{,}000(1-1.04^8)}{1-1.04}$

$\boxed{S_8 \approx \$368{,}569}$

21) A shoe store is closing and wants to sell off its shoes. At the start of the week, the price of all shoes is reduced by 10% of the current price. If a pair of shoes costs \$100 during the first week of the sale, determine the price of those shoes during the 6th week of the sale.

$a_6 = 100(.90)^{6-1}$

$\boxed{a_6 \approx \$59}$

22) A tennis ball is dropped from a height of 150 meters. It rebounds to ¾ the distance from which it fell. How high does it go on its 10th bounce?

$a_1 = 112.5$

$r = .75$

$a_{10} = 112.5(.75)^{10-1}$

$\boxed{a_{10} = 8.45 \text{ meters}}$