Solutions

Skills Check:

1. c
2. 27
3. b
4. 26
5. b
6. 33
7. c
8. *V*
9. a
10. 54
11. b
12. 2600
13. c
14. 6; 8; 10
15. b
16. 9
17. a
18. 18
19. c
20. 16

Cooperative Learning:

Hamiltonian Circuits:

In (1), (3), and (4) there are no Hamiltonian circuits. *ABCDHGEFA* is a Hamiltonian circuit for (2).

Traveling Salesman Problem:

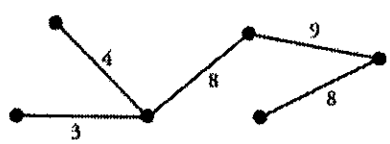
(1) *ADCBA*—33

(2) *ABDCA*—190

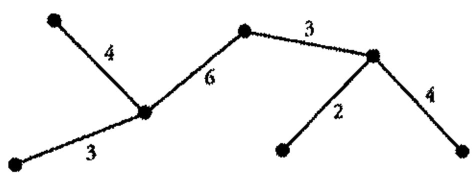
(3) *ABCDEA*—30

Minimum-Cost Spanning Trees:

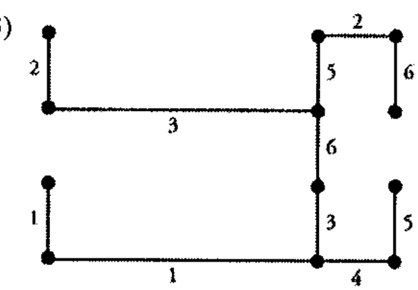
(1)



(2)



(3)



Exercises:

1. **(a)** $X_5X_6X_1X_3X_4X_2X_5$

(b) $X_5X_4X_3X_2X_1X_6X_7X_8X_9X_{10}X_{11}X_{12}X_5$

(c) $X_5X_4X_3X_1X_2X_7X_6X_9X_8X_5$

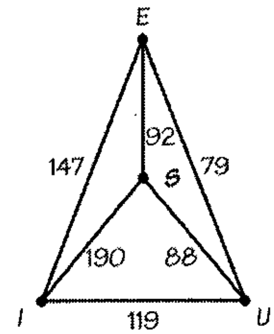
2. **(a)** $X_5X_1X_2X_6X_7X_3X_4X_8X_{12}X_{11}X_{10}X_9X_5$

(b) $X_5X_8X_3X_4X_7X_6X_1X_2X_5$

(c) $X_5X_4X_3X_2X_8X_1X_{10}X_7X_6X_9X_5$

3. Yes, for all three graphs.

37. **(a)**



(b) (1) *UISEU;* mileage $= 119 + 190 + 92 + 79 = 480$
(2) *USIEU;* mileage $= 88 + 190 + 147 + 79 = 504$
(3) *UIESU;* mileage $= 119 + 147 + 92 + 88 = 446$

(c) *UIESU* (Tour 3)

(d) No.

(e) Starting from *U*, one gets *UESIU* Tour 1. From *S* one gets *SUEIS* Tour 2; from *E* one gets *EUSIE* Tour 2; and from *I* one gets *IUESI* Tour 1.

(f) *EUSIE* Tour 2. No.

38. *FRCMF* is quickest and takes 40 minutes.

39. *FMCRF* gets her home in 40 minutes.

40. *FMCR* is quickest and takes 33 minutes.

41. *MACBM* takes 344 minutes to traverse.

42. **(a)** The two methods give identical answers for the first graph. It is *BAEDCB*.
In the second graph nearest neighbor yields *BDAECFB*. While sorted edges yields *BDFCEAB*.

(b) If a complete graph has n vertices, then there are $\frac{(n-1)!}{2}$ Hamiltonian circuits. The first graph would have $\frac{(5-1)!}{2} = \frac{4!}{2} = \frac{4 \times 3 \times 2 \times 1}{2} = \frac{24}{2} = 12$ Hamilton circuits to examine. The second graph would have $\frac{(6-1)!}{2} = \frac{5!}{2} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2} = \frac{120}{2} = 60$ Hamilton circuits to examine.

(c) Answers will vary.

43. A traveling salesman problem.

44. **(a)** *ACBDA* is both the nearest neighbor and sorted edges tour.

(b) Nearest neighbor: *ABCDA* cost 1170; Sorted edge: *ABDCA* cost 1020

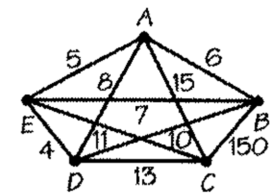
(c) *ADBCEA* is the nearest neighbor tour; *ADEBCA* is the sorted edges tour.

45. A sewer drain inspection route at corners involves finding a Hamiltonian circuit, and there is such a circuit. If the drains are along the blocks, a route in this case involves solving a Chinese postman problem. Since there are 18 odd-valent vertices, an optimal route would require at least 9 reuses of edges. There are many such routes that achieve 9 reuses.

46. **(a)** *AFEDCBA* (from *A*); *BFACDEB* (from *B*)

(b) Sorted edges: *AFEDBCA*

47. The complete graph shown has a different nearest-neighbor tour that starts at A ($AEDBCA$), a sorted-edges tour ($AEDCBA$), and a cheaper tour ($ADBECA$).



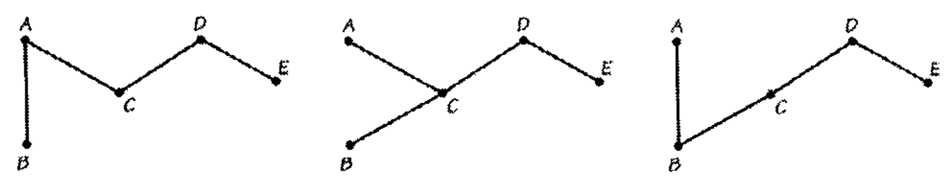
48. There would be $\frac{(20-1)!}{2} = 6.1\times10^{16}$ Hamiltonian circuits whose cost would have to be computed. This would take 1.9 years at a billion tours per second.

49. The optimal tour is the same but its cost is now $4200+10(50)=4700$.

50. **(a)** The graph shown is not a tree because it contains a circuit.
(b) The graph shown is a tree.
(c) The graph shown is not a tree because it contains a circuit.
(d) The graph shown is not a tree because it is not connected.
(e) The graph shown is a tree.
(f) The graph shown is not a tree because it is not connected.
(g) The graph shown is a tree.

51. **(a)** a. Not a tree because there is a circuit. Also, the wiggled edges do not include all vertices of the graph.
b. The circuit does not include all the vertices of the graph.
(b) a. The tree does not include all vertices of the graph.
b. Not a circuit.
(c) a. Not a tree.
b. Not a circuit.
(d) a. Not a tree.
b. Not a circuit.

52. **(a)**



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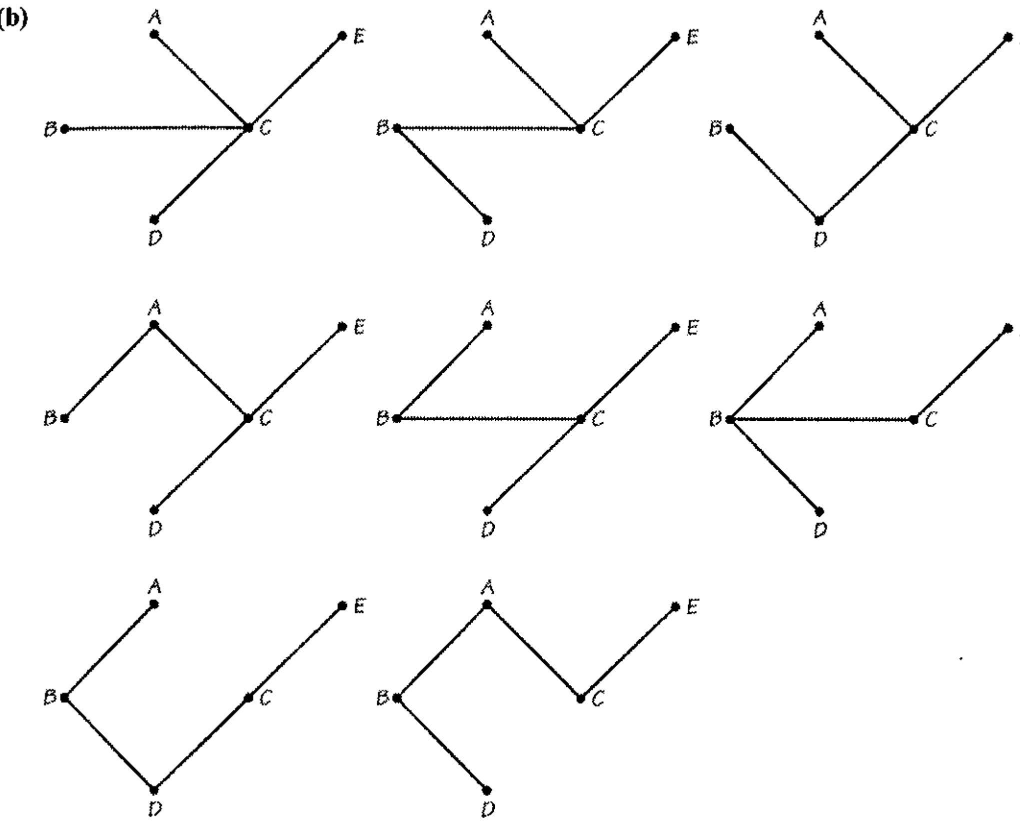
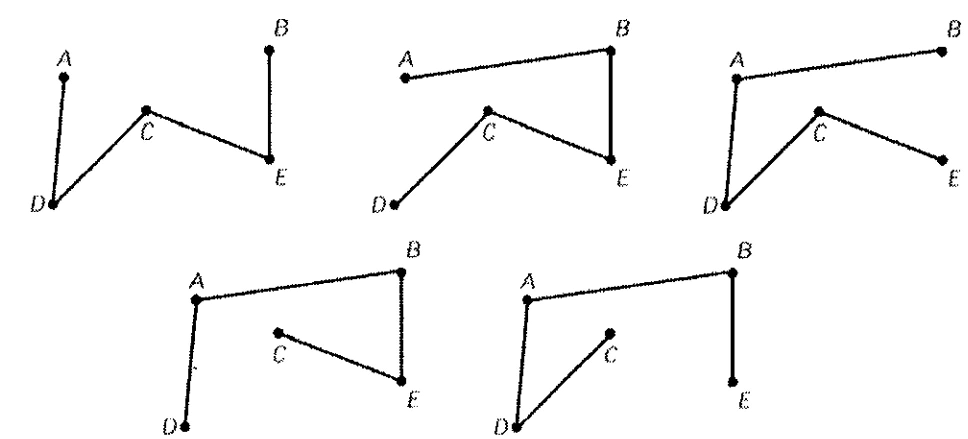
52. continued

(b)



(c)

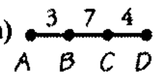


53. **(a)** 1, 2, 3, 4, 5, 8; cost is 23
(b) 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6; cost is 34
(c) 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 6, 7; cost is 60
(d) 1, 2, 2, 3, 3, 3, 4, 5, 5, 5, 6, 6; cost is 45

67. **(a)** Answers will vary for each edge, but the reason it is possible to find such trees is that each edge is an edge of some circuit.

(b) The number of edges in every spanning tree is five, one less than the number of vertices in the graph.

(c) Every spanning tree must include the edge joining vertices C and D, since this edge does not belong to any (simple) circuit in the graph.

68. **(a)** 

(b) The vertices in the graph might represent locations along a road, and the distances between the locations are given by the table shown. Distances along a road would naturally be represented by a graph which is a path. Alternatively, the vertices in the graph might represent manuscripts which were copied by hand from other manuscripts. The weights in the table in this case might represent numbers of key sentences where the manuscripts differ. The graph representing the table in this case being a path suggests that each manuscript was copied from a "prior" manuscript, rather than two manuscripts being copied from one common ancestor, say.

69.

	A	B	C	D
A	0	16	13	5
B	16	0	19	11
C	13	19	0	8
D	5	11	8	0

70. **(a)** The earliest completion time is 37 since the longest path, the unique critical path $T_1 T_4 T_7$ has length 37.

(b) The earliest completion time is 38 since the longest path, the unique critical path $T_1 T_3 T_5 T_8$ has length 38.

71. **(a)** The earliest completion time is 22 since the longest path, the unique critical path $T_3T_2T_5$, has length 22.

(b) The earliest completion time is 30 since the longest path, the unique critical path $T_3T_5T_7$, has length 30.

72.