

Name: \_\_\_\_\_



## LINEAR PROGRAMMING -- ALGEBRA 2

Complete each linear programming problem. Make a labeled graph for each problem and include any other work. List what the variables represent, the constraints (including the hidden ones), the objective function, the vertices, and finally the ordered pair and value of the optimal solution.

### Problem 1: She's Got the "Write" Stuff

Jamie has just finished writing a research paper. She has hired a typist who will type the paper on the computer for her. The typist charges \$3.50 per page if no charts or graphs are used and \$8.00 per page if a chart or graph appears on the page. Jamie knows there will be at most 40 pages having no charts or graphs. There will be no more than 16 pages with charts or graphs, and the paper will be 50 pages or less. What is the greatest possible cost to have the paper typed? How many pages with graphs and how many without graphs would cause this greatest cost?

Variables (in words):  $x =$  No charts  $y =$  charts

Constraints:  $x \leq 40$

$$y \leq 16$$

$$x + y \leq 50$$

$$x \geq 0$$

$$y \geq 0$$

Objective Function: Find Maximum Cost

$$C(x,y) = 3.50x + 8y$$

Vertices: of Feasible Region:

$$(0,0) \quad 0 \quad (40,10) \quad 220$$

$$(0,16) \quad 128 \quad (40,0) \quad 140$$

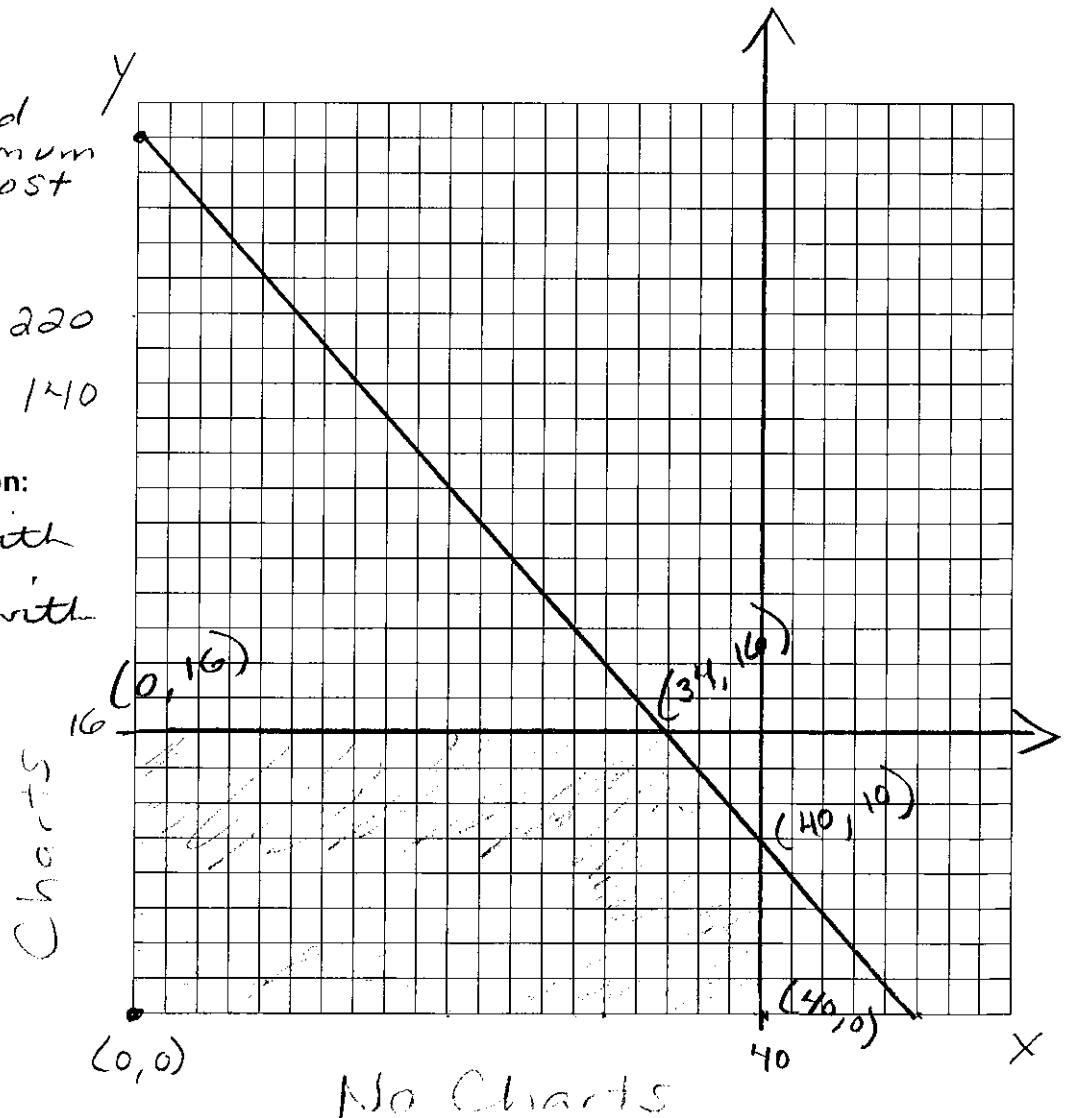
$$(34,16) \quad 247$$

Ordered Pair of Optimal Solution:

$$(34,16) \quad \begin{array}{l} 34 \text{ pages with} \\ \text{no charts,} \\ 16 \text{ pages with} \\ \text{charts} \end{array}$$

Maximum Cost of the Paper:

$$\$ 247$$





## Problem 2: Batter Up

BingBATABoom, Inc manufactures two different quality wood baseball bats, the *Battlefield* and the *Dingbat*. The *Battlefield* takes 8 hours to trim and turn and 2 hours to finish it. It has a profit of \$17. The *Dingbat* takes 5 hours to trim and turn and 5 hours to finish, but its profit is \$29. The total time per day available for trimming and turning is 80 hours and for finishing is 50 hours. How many of each type of bat should be produced to have the maximum profit? What is this maximum profit?

Variables (in words):  $x =$  Battlefield  $y =$  Dingbat

	Trim	Finish	
x	8	2	\$ 17
y	5	5	\$ 29
	80	50	

Constraints:  $8x + 5y \leq 80$

x	y
0	16
10	0

$2x + 5y \leq 50$

$x \geq 0$

$y \geq 0$

x	y
0	10
25	0

Objective Function: Maximize Profit

$P(x,y) = 17x + 29y$

Vertices: of Feasible Region:

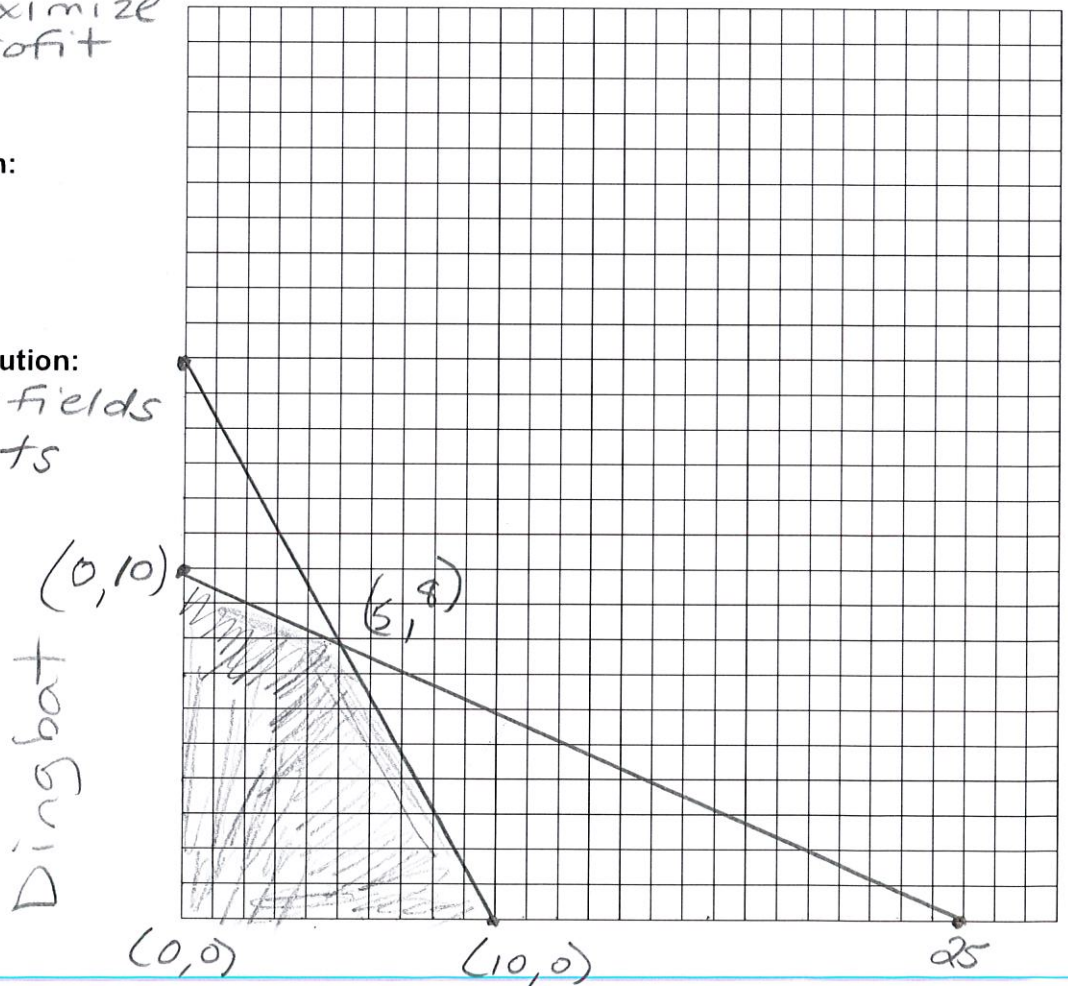
- $(0,0)$  0
- $(0,10)$  290
- $(5,8)$  317
- $(10,0)$  170

Ordered Pair of Optimal Solution:

- $(5,8)$  5 Battlefields
- 8 Dingbats

Maximum Profit:

\$ 317



Battlefield

**Problem 3: Get the CARrect Answer**

$$20(50)(.6) = 600 \text{ m}^2 \text{ usable space}$$



The Gala Events Center has a rectangular parking lot measuring 20 m by 50 m. Only 60% of the lot is usable space. A car requires 6 square meters of space and a bus requires 30 square meters of space. The attendant can handle no more than 60 vehicles. If the parking fees are \$2.50 for cars and \$7.50 for buses, how many of each type of vehicle should the attendant accept to maximize income? What is the maximum income?

Variables (in words):  $x = \text{Cars}$        $y = \text{buses}$

	Space	vehicles
x	6 m <sup>2</sup>	x
y	30 m <sup>2</sup>	y
	600 m <sup>2</sup>	60

Constraints:

$$6x + 30y \leq 600$$

$$x + y \leq 60$$

$$x \geq 0$$

$$y \geq 0$$

Objective Function: Maximize Profit

x	y
0	20
100	0

$$P(x,y) = 2.50x + 7.50y$$

Vertices: of Feasible Region:

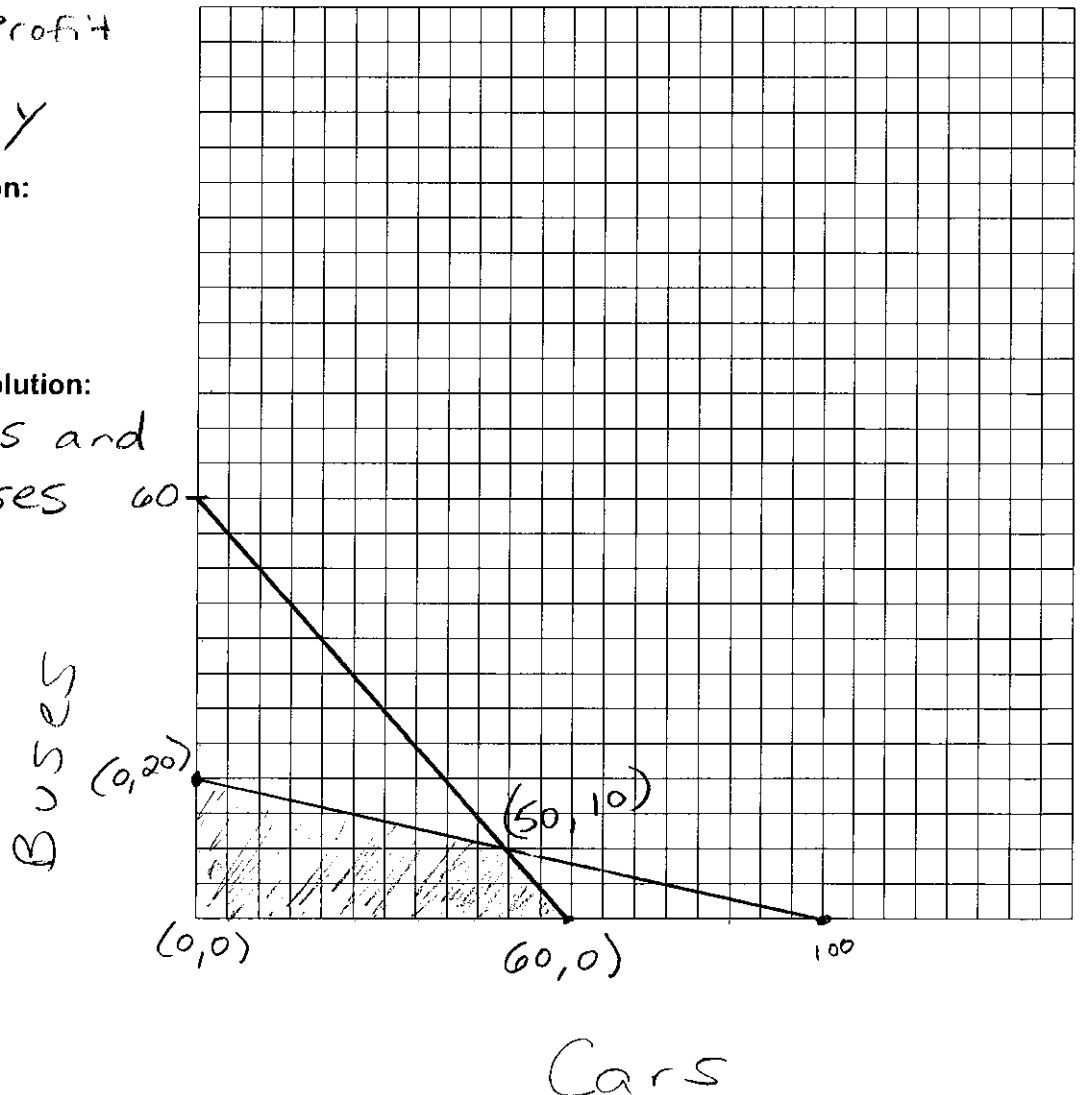
- (0,0)      0
- (0,20)    150
- (50,10)   200
- (60,0)    150

Ordered Pair of Optimal Solution:

(50,10)    50 cars and  
10 buses

Maximum Income:

\$200





### Problem 4: Close to "Nuttin"

At "Nuttin' Like a Lighting Bolt Manufacturing" the cost to run Machine 1 for one hour is \$2.00. During that hour, Machine 1 produces 240 bolts and 100 nuts. The cost to run Machine 2 for an hour is \$2.40. During that hour, Machine 2 produces 160 bolts and 160 nuts. With a combined running time of no more than 30 hours, how long should each machine run to produce an order of at least 2080 bolts and 1520 nuts at the minimum operating cost?

Variables (in words):  $x = \text{Machine 1}$   $y = \text{Machine 2}$

	Bolts	Nuts	Cost
Machine 1	240	100	2.00
Machine 2	160	160	2.40
	$\geq 2080$	$\geq 1520$	

Constraints:

$$x + y \leq 30$$

$$240x + 160y \geq 2080$$

$$100x + 160y \geq 1520$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 30 \\ 30 & 0 \end{array}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 13 \\ 8.7 & 0 \end{array}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 9.5 \\ 15.2 & 0 \end{array}$$

Objective Function: Minimum Cost

$$C(x,y) = 2x + 2.4y$$

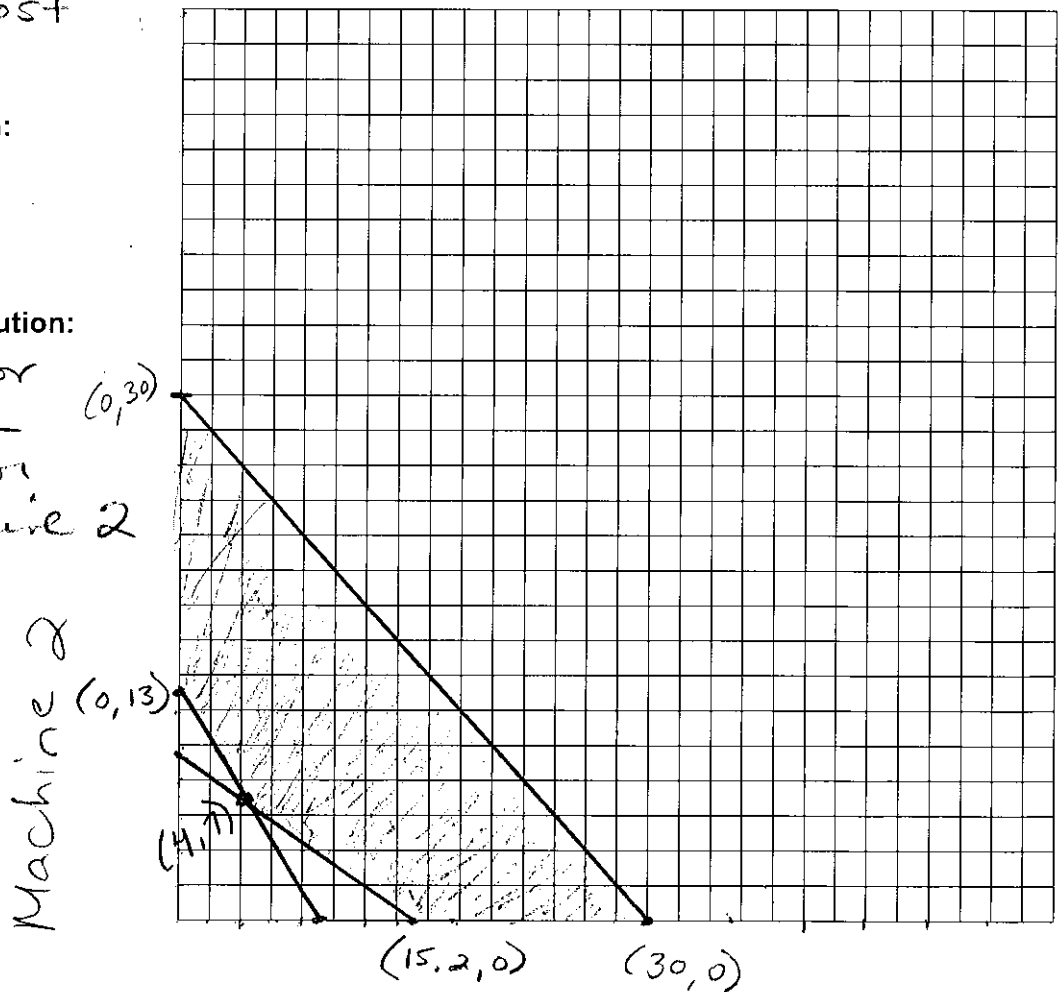
Vertices: of Feasible Region:

- $(0, 30)$  72
- $(0, 13)$  31.20
- $(15.2, 0)$  30.40
- $(30, 0)$  60
- $(4, 7)$  24.80

Ordered Pair of Optimal Solution:

$(4, 7)$  4 hours for Machine 1  
7 hours for Machine 2  
Minimum Cost: Machine 2

~~60~~ 24.80



Machine 1

### Problem 5: Situation for a "Fungi"

A biologist is developing two new strains of bacteria. Each sample of Type I bacteria produces 4 new viable bacteria and each sample of Type II produces 3 new viable bacteria. Altogether, at least 240 new viable bacteria must be produced. At least 30, but no more than 60, of the original samples must be Type I. No more than 70 of the samples can be Type II. A sample of Type I cost \$7 and a sample of Type II costs \$3. How many samples of each should be used to minimize the cost? What is the minimum cost?

Variables (in words):  $x = \text{Type I}$        $y = \text{Type II}$   
 $C = 7$                                        $C = 3$

Constraints:  $4x + 3y \geq 240$        $\begin{array}{c|c} x & y \\ \hline 0 & 80 \\ 60 & 0 \end{array}$   
 $30 \leq x \leq 60$   
 $y \leq 70$

Objective Function: Minimize Cost

$$C(x, y) = 7x + 3y$$

Vertices: of Feasible Region:

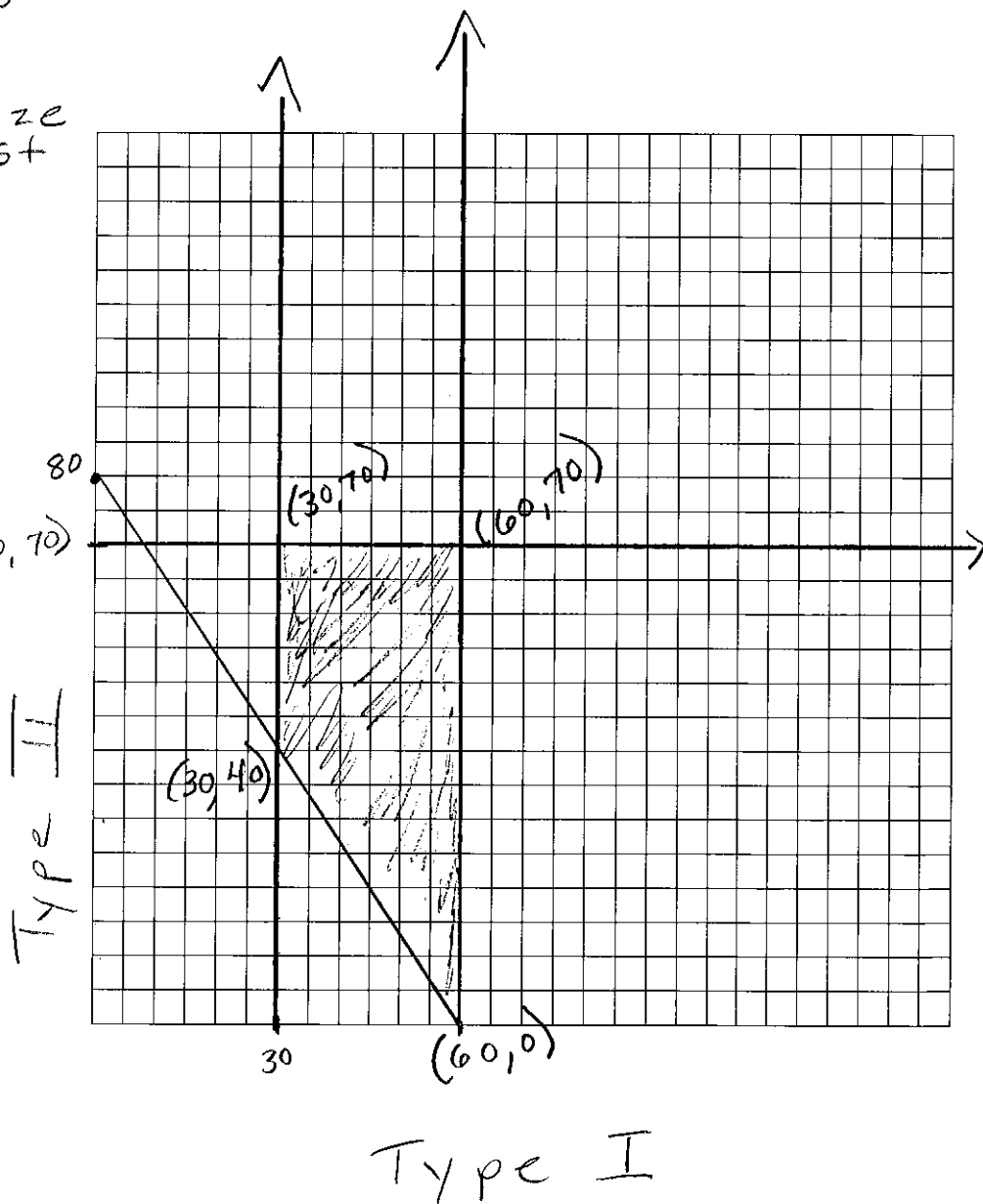
$(20, 70)$       420  
 $(0, 70)$       630  
 $(60, 0)$       420  
 $(30, 40)$       330

Ordered Pair of Optimal Solution:

$(30, 40)$       30 Type I  
 and 40 Type II  $(0, 70)$

Minimum Cost:

\$ 330





### Problem 6: "Fat"astic Meals

A school dietitian wants to prepare a meal of meat and vegetables that have the lowest possible fat and that meet the Food and Drug Administration recommended daily allowances (RDA) of iron and protein. The RDA minimums are 24 milligrams of iron and 50 grams of protein. Each serving of meat contains 10 grams of protein, 4 milligrams of iron, and 5 grams of fat. Each serving of vegetables contains 5 grams of protein, 6 milligrams of iron, and 3 grams of fat. Write an objective function for the number of grams of fat, and find the minimum number of grams of fat. What is the minimum number of grams of fat?

Variables (in words):  $x = \text{Meat}$        $y = \text{Vegetables}$

Constraints:

$$4x + 6y \geq 24$$

$$10x + 5y \geq 50$$

		Iron	Protein	Fat
x	Meat	4	10	5
y	veg	6	5	3
		24	50	

Objective Function: Lowest Fat

$$F(x,y) = 5x + 3y$$

Vertices: of Feasible Region:

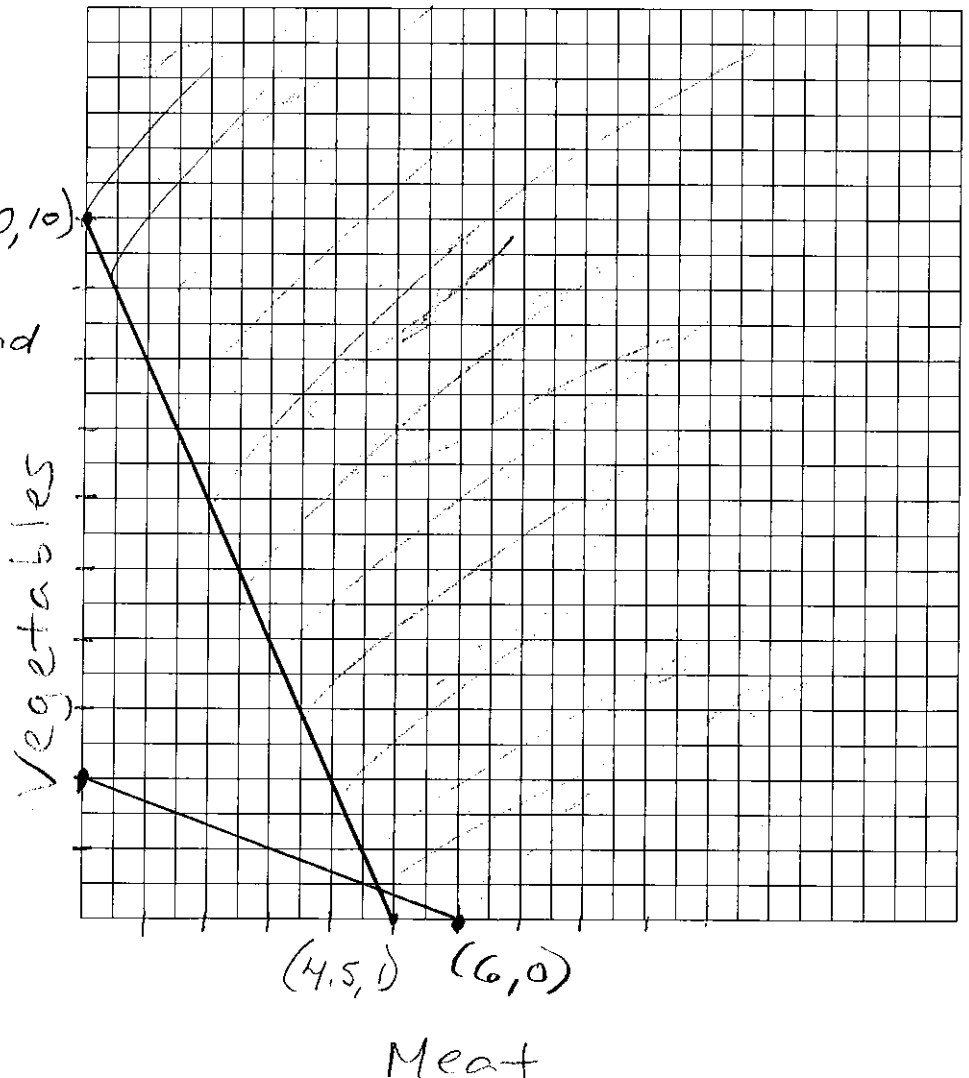
- $(0, 10)$       30
- $(4.5, 1)$       25.5
- $(6, 0)$       30

Ordered Pair of Optimal Solution:

- $(4.5, 1)$       4.5 meat and 1 vegetable

Minimum Fat Grams:

25.5





### Problem 7: Be a Doll and Solve This



One of the dolls that *Barbiehoo-R-Us* manufactures is the *Talking Tommy*. Another doll without this talking mechanism is called *Silent Sally*. In one hour, the company can produce 8 *Talking Tommy* dolls or 20 *Silent Sally* dolls. Because of the demand, the company knows that it must produce at least twice as many *Talking Tommy* dolls as the *Silent Sally* dolls. The company spends no more than 48 hours per week making these two dolls. The profit on each *Talking Tommy* is \$3.00 and the profit on each *Silent Sally* is \$7.50. How many of each doll should be produced to maximize profit each week? What is this profit?

Variables (in words):  $x = TT$        $y = SS$

		1 hour	P
x	TT	8	3
y	SS	20	7.5
		48	

Constraints:

$$y < \frac{1}{2}x$$

$$x + y \leq 48$$

x	y
0	0
1	2

Objective Function:

$$P(x,y) = 3(8)x + 7.5(20)y \quad (0, 48)$$

Vertices: of Feasible Region:

- (0, 0)      0
- (48, 0)      1152
- (32, 16)      3168

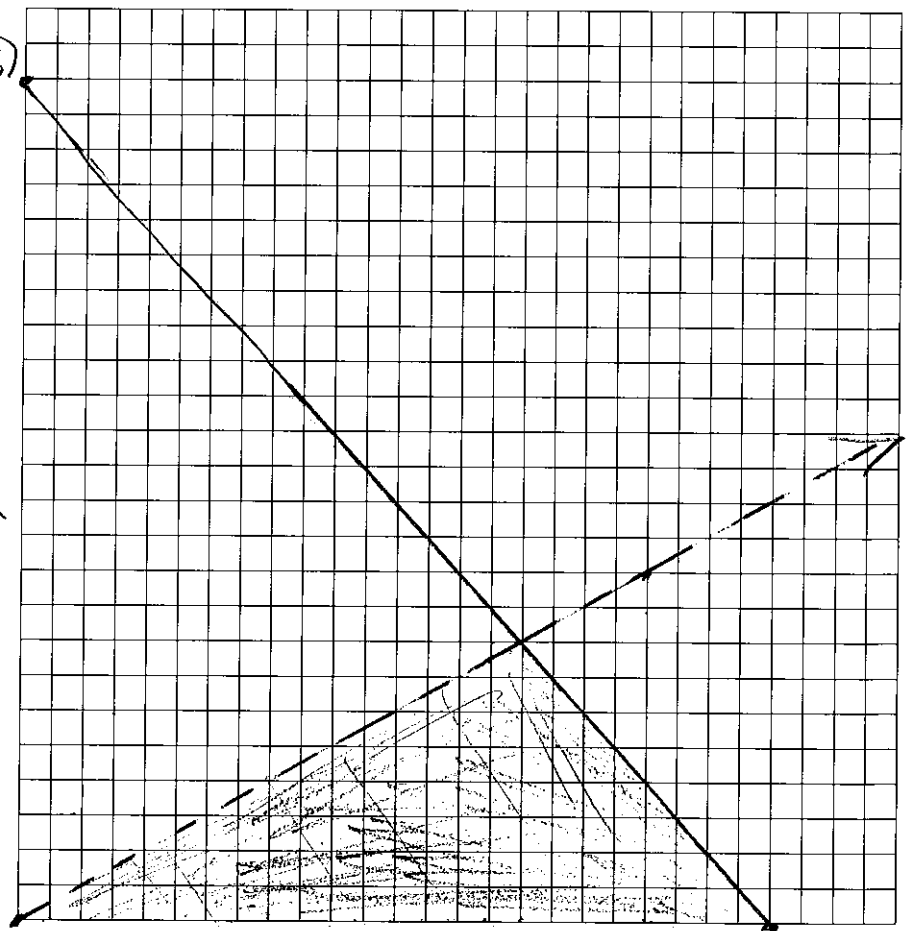
Ordered Pair of Optimal Solution:

(32, 16)      32 Talking Tommy and

Maximum Profit: 16 Silent Sally

\$3168

Silent Sally



Talking Tommy (48, 0)