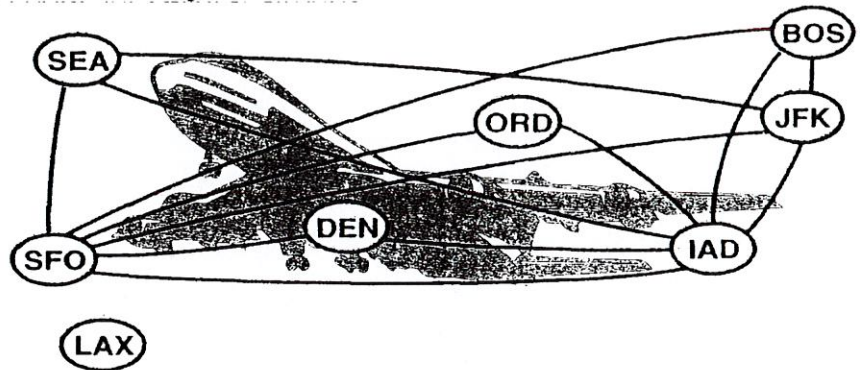


The map of the airline's route system that you examined in sheet 1 is rather elaborate and contains many cities and flights. To understand better how graph theory can be applied to solving problems in the airline industry, a few cities will be examined in more detail. Consider the representation shown in the graph below for an airline called "Fast Flyers."

The following chart lists the standard codes for several airports:

City	Code
Boston, Massachusetts	BOS
Chicago, Illinois (O'Hare)	ORD
Denver, Colorado	DEN
Los Angeles, California	LAX
New York City (Kennedy)	JFK
San Francisco, California	SFO
Seattle, Washington	SEA
Washington, D.C. (Dulles Airport)	IAD



- Briefly describe what the graph represents.
- The graph shows paths from one city to another. For example, one path from Washington, D.C., to San Francisco is {IAD, ORD, SFO}. Notice that the vertices are listed in a specific order. List at least four other paths from Washington, D.C., to San Francisco.
- A connected graph is a graph such that for every two vertices  $a$  and  $b$ , a path exists from  $a$  to  $b$ .
  - Is the graph a connected graph? If not, name a city that is not connected with the rest of the graph.
  - Revise the graph by adding the edges {LAX, SFO} and {LAX, IAD} to it. Is it then a connected graph? Are both these edges needed to make the graph connected? Why or why not? Use this revised version of the graph for the remainder of the activity.
- The degree of a vertex is the number of edges that are incident to, that is, connected with, that vertex.
  - What city in the graph has the highest degree? What is that city's degree?
  - What would a degree of 1 mean about flying to that city?
- How many edges are in the revised graph above? What is the sum of the degrees of the vertices? Can you make a conjecture about a relationship between the number of edges and the sum of the degrees of the vertices? Explain why this relationship seems reasonable.
- A graph is called a complete graph if all possible edges between pairs of vertices are included in the graph. If the graph above was redrawn as a complete graph, what would be the degree of each vertex? How many edges would it have? Explain your answer and your methods of solution. Include at least one method that does not involve counting all edges. Draw the graph as a complete graph.
- If you are managing Fast Flyers' operations, do you think offering direct flights between all cities to which it flies is a good idea?