NAME ________________________ DATE ____________ PERIOD ______

D 12 Study Guide and Intervention

The Binomial Theorem

Pascal's Triangle **Pascal's triangle** is the pattern of coefficients of powers of binomials displayed in triangular form. Each row begins and ends with 1 and each coefficient is the sum of the two coefficients above it in the previous row.

Pascal's Triangle	$(a + b)^0$						1					
	$(a + b)^1$					1		1				
	$(a + b)^2$				1		2		1			
	$(a + b)^3$			1		3		3		1		
	$(a + b)^4$		1		4		6		4		1	
	$(a + b)^5$	1		5		10		10		5		1

Example **Use Pascal's triangle to find the number of possible sequences consisting of 3 *a*s and 2 *b*s.**

The coefficient 10 of the a^3b^2-term in the expansion of $(a + b)^5$ gives the number of sequences that result in three *a*s and two *b*s.

Exercises

Expand each power using Pascal's triangle.

1. $(a + 5)^4$

2. $(x - 2y)^6$

3. $(j - 3k)^5$

4. $(2s + t)^7$

5. $(2p + 3q)^6$

6. $\left(a - \frac{b}{2}\right)^4$

7. Ray tosses a coin 15 times. How many different sequences of tosses could result in 4 heads and 11 tails?

8. There are 9 true/false questions on a quiz. If twice as many of the statements are true as false, how many different sequences of true/false answers are possible?

D 12 Study Guide and Intervention (continued)

The Binomial Theorem

The Binomial Theorem

Binomial Theorem	If n is a nonnegative integer, then $(a + b)^n = 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \ldots + 1a^0b^n$

Another useful form of the Binomial Theorem uses **factorial** notation and sigma notation.

Factorial	If n is a positive integer, then $n! = n(n-1)(n-2) \cdot \ldots \cdot 2 \cdot 1$.
Binomial Theorem, Factorial Form	$(a + b)^n = \frac{n!}{n!0!}a^nb^0 + \frac{n!}{(n-1)!1!}a^{n-1}b^1 + \frac{n!}{(n-2)!2!}a^{n-2}b^2 + \ldots + \frac{n!}{0!n!}a^0b^n$ $= \sum_{k=0}^{n} \frac{n!}{(n-k)!k!}a^{n-k}b^k$

Example 1 **Evaluate $\frac{11!}{8!}$.**

$$\frac{11!}{8!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$= 11 \cdot 10 \cdot 9 = 990$$

Example 2 **Expand $(a - 3b)^4$.**

$$(a - 3b)^4 = \sum_{k=0}^{4} \frac{4!}{(4-k)!k!}a^{4-k}(-3b)^k$$
$$= \frac{4!}{4!0!}a^4 + \frac{4!}{3!1!}a^3(-3b)^1 + \frac{4!}{2!2!}a^2(-3b)^2 + \frac{4!}{1!3!}a(-3b)^3 + \frac{4!}{0!4!}(-3b)^4$$
$$= a^4 - 12a^3b + 54a^2b^2 - 108ab^3 + 81b^4$$

Exercises

Evaluate each expression.

1. $5!$

2. $\frac{9!}{7!2!}$

3. $\frac{10!}{6!4!}$

Expand each power.

4. $(a - 3)^6$

5. $(r + 2s)^7$

6. $(4x + y)^4$

7. $\left(2 - \frac{m}{2}\right)^5$

Find the indicated term of each expansion.

8. third term of $(3x - y)^5$

9. fifth term of $(a + 1)^7$

10. fourth term of $(j + 2k)^8$

11. sixth term of $(10 - 3t)^7$

12. second term of $\left(m + \frac{2}{3}\right)^9$

13. seventh term of $(5x - 2)^{11}$