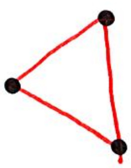
Complete Graphs

A **complete graph** is a graph in which every vertex is adjacent to every other vertex in the graph.

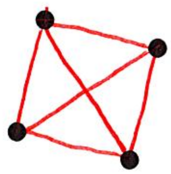
We use: $\boxed{K_N}$ to represent "The complete graph with N vertices."

Draw:

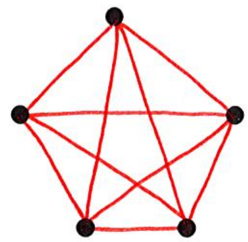
K_3



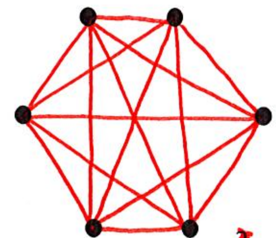
K_4



K_5



K_6



In a Complete Graph with N vertices:

Each vertex has degree:	$N-1$
Total number of edges:	$\frac{N(N-1)}{2}$
Total number of Hamilton Circuits: (including Mirror-Images)	$(N-1)!$

As the number of edges increases, the number of Hamilton Circuits GREATLY increases. If we are looking for "the best" Hamilton Circuit for our traveling salesman, we need to have a plan!

Example:
How many Hamilton Circuits are there in:

1. K_3 $=(3-1)! = 2! = 2\cdot 1 = 2$
2. $K_4 = (4-1)! = 3! = 3\cdot 2\cdot 1 = 6$
3. $K_5 = (5-1)! = 4! = 4\cdot 3\cdot 2\cdot 1 = 24$
4. $K_6 = (6-1)! = 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$
5. $K_{10} = (10-1)! = 9! = 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$ $\boxed{362{,}880}$

Example: How many edges are there in

1. K_{19}

$\frac{19(19-1)}{2} = \boxed{171}$

2. K_{150}

$\frac{150(150-1)}{2} = \boxed{11{,}175}$

Example: How many vertices (what is N) if

3. K_N has 5040 Hamilton Circuits?

Guess and check.

$\boxed{N=8}$

$(8-1)! = 7!$

$7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = \boxed{5040}$

4. K_N has 990 edges ?

$990 = \frac{n(n-1)}{2}$

$1980 = n^2 - n$

$0 = n^2 - n - 1980$

Use quadratic formula, factor or calculator to solve

$\boxed{n=45}$

5. K_N has 120 Hamilton Circuits ?

Guess and check

$\boxed{N=6}$

$(6-1)! = 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

$\boxed{120}$

6. K_N has 136 edges?

$136 = \frac{n(n-1)}{2}$

$272 = n^2 - n$

$0 = n^2 - n - 272$

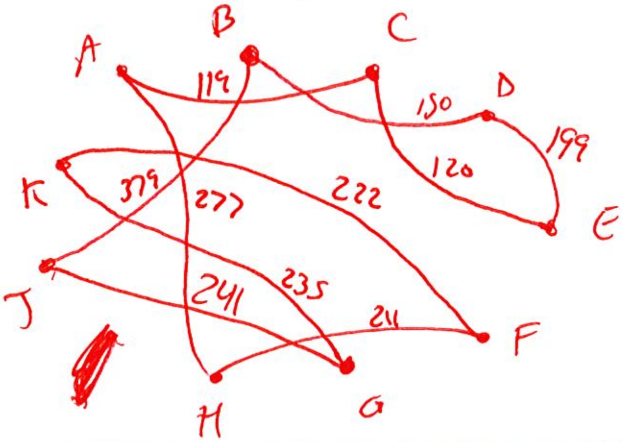
Use quadratic formula, factor or calculator to solve

$\boxed{n=17}$

A much more practical approach is to organize the data in a table.

	A	B	C	D	E	F	G	H	J	K
A	*	185	119	152	133	321	297	277	412	381
B	185	*	121	150	200	404	458	492	379	427
C	119	121	*	174	120	332	439	348	245	443
D	152	150	174	*	199	495	480	500	454	489
E	133	200	120	199	*	315	463	204	396	487
F	321	404	332	495	315	*	356	211	369	222
G	297	458	439	480	463	356	*	471	241	235
H	277	492	348	500	204	211	471	*	283	478
J	412	379	245	454	396	369	241	283	*	304
K	381	427	443	489	487	222	235	478	304	*

Use the nearest neighbor algorithm to find the Hamilton Circuit starting at vertex A.



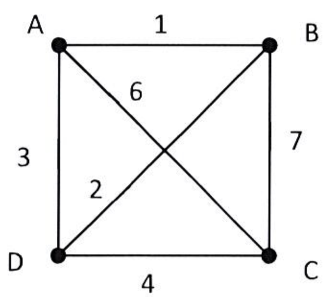
NN = 2063

A – C – E – D – B – J – G – K – F – H – A

Example:

a) Use the nearest neighbor method to find the Hamilton Circuit starting with vertex A.

b) Use the nearest neighbor method to find the Hamilton Circuit starting with vertex B.



A – B – D – C – A = 13

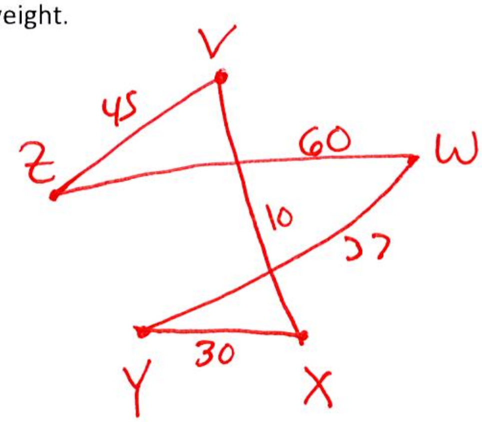
B – A – D – C – A = 14

Example: The chart below represents the price of a bus ticket between the following cities.

You would like to start and end at your hometown of Z.

Find a Hamilton Circuit and its weight.

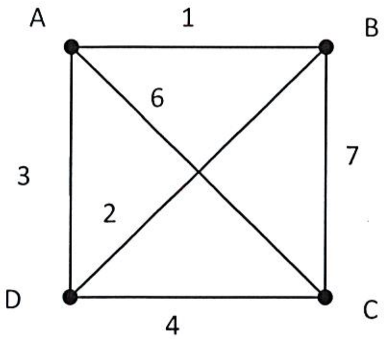
	V	W	X	Y	Z
V	**	$35	$10	$20	$45
W	$35	**	$42	$37	$60
X	$10	$42	**	$30	$50
Y	$20	$37	$30	**	$77
Z	$45	$60	$50	$77	**



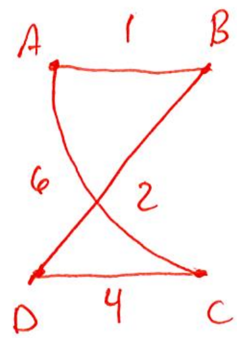
Z – V – X – Y – W – Z = 182

Examples: Use the Cheapest Link Method to find a Hamilton Circuit.

1. Home City A

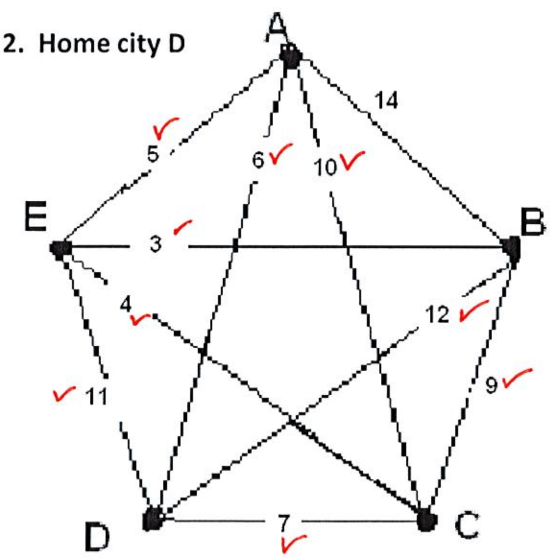


AB = 1 ✓
BD = 2 ✓
AD = 3 X
DC = 4 ✓
AC = 6 ✓
BC = 7

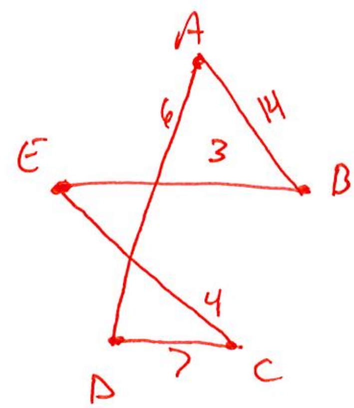


A - B - D - C - A = 13

2. Home city D



EB = 3 ✓
EC = 4 ✓
EA = 5 X
AD = 6 ✓
DC = 7 ✓
BC = 9 X
AC = 10 X
ED = 11 X
BD = 12 X
AB = 14

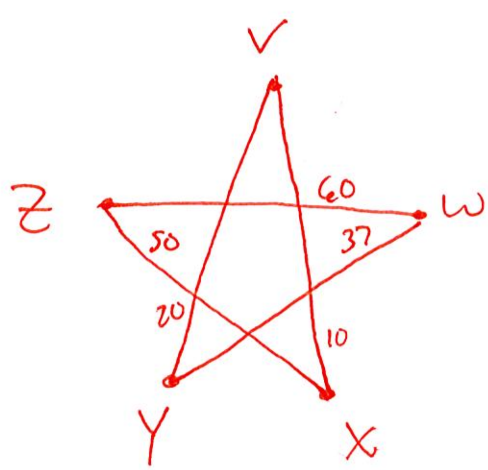


D - A - B - E - C - D = 34

3. Home City Z

	V	W	X	Y	Z
V	**	$35	$10	$20	$45
W	$35	**	$42	$37	$60
X	$10	$42	**	$30	$50
Y	$20	$37	$30	**	$77
Z	$45	$60	$50	$77	**

XV = 10 ✓
VY = 20 ✓
XY = 30 X
WV = 35 X
WY = 37 ✓
WX = 42 X
VZ = 45 X
XZ = 50 ✓
WZ = 60
YZ = 77



Z - X - V - Y - W - Z = 177