

Complete Graphs

A complete graph is a graph in which every vertex is adjacent to every other vertex in the graph.

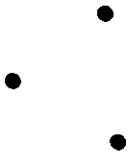
We use:



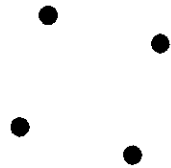
to represent "The complete graph with N vertices."

Draw:

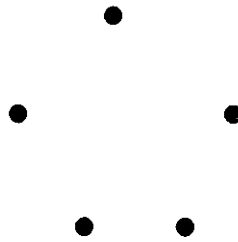
K_3



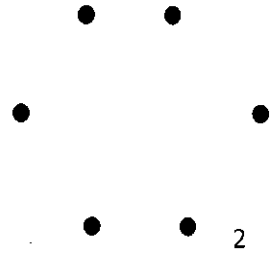
K_4



K_5



K_6



In a Complete Graph with N vertices:Each vertex has degree: $N - 1$ Total number of edges: $\frac{N(N-1)}{2}$ Total number of Hamilton Circuits:
(including Mirror-Images) $(N-1)!$ **Example:**

How many Hamilton Circuits are there in:

1. K_3 2. K_4 3. K_5 4. K_6 5. K_{10}

As the number of edges increases, the number of Hamilton Circuits GREATLY increases. If we are looking for "the best" Hamilton Circuit for our traveling salesman, we need to have a plan!

Example: How many edges are there in1. K_{19} 2. K_{150} **Example:** How many vertices (what is N) if3. K_N has 5040 Hamilton Circuits?5. K_N has 120 Hamilton Circuits ?4. K_N has 990 edges ?6. K_N has 136 edges?

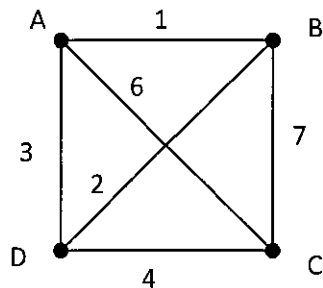
A much more practical approach is to organize the data in a table.

	A	B	C	D	E	F	G	H	J	K
A	*	185	119	152	133	321	297	277	412	381
B	185	*	121	150	200	404	458	492	379	427
C	119	121	*	174	120	332	439	348	245	443
D	152	150	174	*	199	495	480	500	454	489
E	133	200	120	199	*	315	463	204	396	487
F	321	404	332	495	315	*	356	211	369	222
G	297	458	439	480	463	356	*	471	241	235
H	277	492	348	500	204	211	471	*	283	478
J	412	379	245	454	396	369	241	283	*	304
K	381	427	443	489	487	222	235	478	304	*

Use the nearest neighbor algorithm to find the Hamilton Circuit starting at vertex A.

Example:

- a) Use the nearest neighbor method to find the Hamilton Circuit starting with vertex A.
- b) Use the nearest neighbor method to find the Hamilton Circuit starting with vertex B.

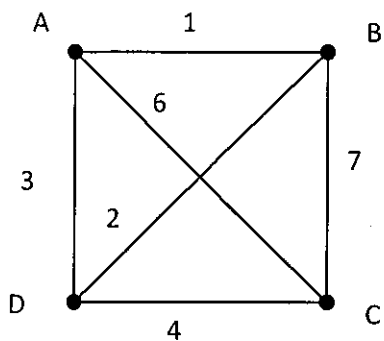


Example: The chart below represents the price of a bus ticket between the following cities. You would like to start and end at your hometown of Z. Find a Hamilton Circuit and its weight.

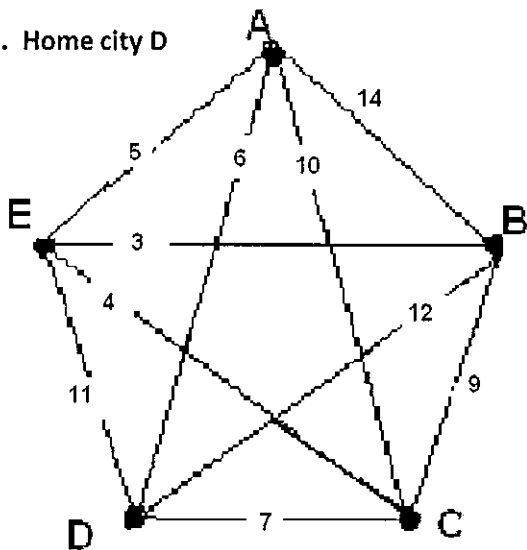
	V	W	X	Y	Z
V	**	\$35	\$10	\$20	\$45
W	\$35	**	\$42	\$37	\$60
X	\$10	\$42	**	\$30	\$50
Y	\$20	\$37	\$30	**	\$77
Z	\$45	\$60	\$50	\$77	**

Examples: Use the Cheapest Link Method to find a Hamilton Circuit.

1. Home City A



2. Home city D



3. Home City Z

	V	W	X	Y	Z
V	**	\$35	\$10	\$20	\$45
W	\$35	**	\$42	\$37	\$60
X	\$10	\$42	**	\$30	\$50
Y	\$20	\$37	\$30	**	\$77
Z	\$45	\$60	\$50	\$77	**