

Key

1. Arrow's Impossibility Theorem states that any voting system can give undesirable outcomes. Explain what this means. *No voting system is perfect. All voting systems have flaws.*

2. There are 18 delegates to a political party's convention at which four people, A, B, C, and D, have been nominated as the party's candidate for governor. The delegates' preference schedule is shown below.

|                 | Number of Delegates |   |   |
|-----------------|---------------------|---|---|
| 3 First choice  | 8                   | 9 | 4 |
| 2 Second choice | A                   | B | C |
| 1 Third choice  | B                   | A | B |
| 0 Fourth choice | C                   | D | A |
|                 | D                   | C | D |

*Handwritten calculations:*

|               | A  | B  | C  | D |
|---------------|----|----|----|---|
| First choice  | 24 | 16 | 9  | 4 |
| Second choice | 18 | 27 | 12 | 0 |
| Third choice  | 4  | 8  | 0  | 0 |
| Fourth choice | 0  | 0  | 0  | 0 |

A. What nominee would be elected if the party uses a Borda count?

B

B. What Candidate would win using the Hare Voting System?

B

C. Which Candidate will win using Sequential Pairwise Voting

*Agenda B-C-D-A*

B

*Handwritten pairwise comparisons:*

| BC             | BD             | BA             |
|----------------|----------------|----------------|
| 8   4<br>9   0 | 5   4<br>9   0 | 9   8<br>4   0 |

*Results: B wins BC, B wins BD, B wins BA.*

*Handwritten Borda counts:*

A = 8  
B = 9  
C = 4

*Handwritten Hare counts:*

A = 8  
B = 13

3. After their star pitcher moved to another town, the eight remaining members of the company baseball team needed to select a new pitcher. They used approval voting on the four prospects, and the results are listed below. An "X" indicates an approval vote.

|       |   |   |   |   |   |
|-------|---|---|---|---|---|
| Alan  | X | X |   |   | X |
| Bob   | X |   | X | X | X |
| Chuck |   | X |   | X | X |
| David |   | X | X |   | X |

Which pitcher is chosen if just one is to be selected?

Bob

4. An 11-member committee must choose one of the four applicants, K, L, M, and N, for membership on the committee. The committee members have preferences among the applicants as given below.

|               | Number of Members |   |   |             |             |             |
|---------------|-------------------|---|---|-------------|-------------|-------------|
|               | 6                 | 2 | 3 | K/M         | K/L         | L/M         |
| First choice  | K                 | M | M | 6<br>2<br>3 | 6<br>2<br>3 | 6<br>2<br>3 |
| Second choice | L                 | L | N |             |             |             |
| Third choice  | N                 | K | L |             |             |             |
| Fourth choice | M                 | N | K |             |             |             |

*Handwritten notes:* In the K/M comparison, K has 6 votes and M has 2+3=5 votes, with K circled. In the K/L comparison, K has 6 votes and L has 2+3=5 votes, with K circled. In the L/M comparison, L has 6 votes and M has 2+3=5 votes, with L circled.

- A. Is there a Condorcet Winner? If so, who?

*Handwritten answer:* Yes, K ~~wins~~ wins against all opponents.

5. Eight board members vote by approval voting on four candidates, A, B, C, and D, for new positions on their board as indicated in the following table. An "X" indicates an approval vote.

|   | Voters |   |   |   |   |   |   |   |
|---|--------|---|---|---|---|---|---|---|
|   | 1      | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | X      | X | X | X |   | X | X | X |
| B |        | X | X |   | X | X |   | X |
| C |        | X |   | X | X |   | X | X |
| D | X      |   | X | X | X | X | X |   |

- A. Which candidate will be chosen for the board if just one of them is to be elected?

*Handwritten answer:* A

- B. Which candidate(s) is (are) elected if 80% approval is necessary and at most two are elected?

*Handwritten answer:* A